THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Solutions to Assignment 2

MATH1111: Introduction to Calculus — Semester 1, 2008

1. Consider $f(x) = -4x^2 + x - 1$.

(a) Find $f'(x)$.

Solution: $f'(x) = -8x + 1$.
1 Mark

(b) Find the point on the graph $y = f(x)$ where $f'(x) = 0$.

Solution: $f'(x) = -8x + 1 = 0 \Rightarrow x = \frac{1}{8}$. And $f(\frac{1}{8}) = -\frac{15}{16}$. So the point for which $f'(x) = 0$ is $(\frac{1}{8}, -\frac{15}{16})$.
2 Marks: 1 for $x = 1/8$ and 1 for the point $(\frac{1}{8}, -\frac{15}{16})$.

(c) Find $f''(x)$.

Solution: $f''(x) = -8$.
1 Mark

(d) Using your result from part c) explain why the graph of $y = f(x)$ is concave down.

Solution: $f(x)$ is concave down as $f''(x) < 0$ for all x.
1 Mark

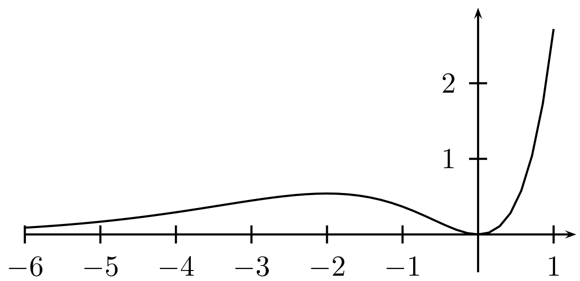
2. Find the equation of the line tangent to the graph of f at $(\pi, 0)$, where f is given by

$$f(x) = x^3 \sin(2x).$$

Solution: $f'(x) = 3x^2 \sin(2x) + 2x^3 \cos(2x)$. So $f'(\pi) = 2\pi^3$ and the equation of the line tangent at $(\pi, 0)$ is $y = 2\pi^3(x - \pi)$.
3 Marks: 1 for $f'(x)$, 1 for $f'(\pi)$ and 1 for the equation of the tangent line.

3. Find all the local maximum, local minimum and inflection points of $f(x) = x^2 e^x$ and hence sketch $y = f(x)$ showing all the points you've found.

Solution: $f'(x) = 2xe^x + x^2e^x = x(2+x)e^x$ and $f''(x) = 2e^x + 4xe^x + x^2e^x = (x^2 + 4x + 2)e^x$. $f'(x) = 0 \Rightarrow x = 0, -2$. $f''(0) > 0$ and $f(0) = 0$ so $(0, 0)$ is a local minimum. $f''(-2) < 0$ and $f(-2) = 4e^{-2}$ so $(-2, 4e^{-2})$ is a local maximum.
$f''(x) = 0 \Rightarrow x = \dfrac{-4 \pm \sqrt{16 - 4 \times 2}}{2} = -2 \pm \sqrt{2}$. $f(-2 + \sqrt{2}) = (-2 + \sqrt{2})^2 e^{-2+\sqrt{2}}$ and $f(-2 - \sqrt{2}) = (-2 - \sqrt{2})^2 e^{-2-\sqrt{2}}$, so the points of inflection are $(-2 + \sqrt{2}, (-2 + \sqrt{2})^2 e^{-2+\sqrt{2}})$ and $(-2 - \sqrt{2}, (-2 - \sqrt{2})^2 e^{-2-\sqrt{2}})$ or approximately $(-0.6, 0.2)$ and $(-3.4, 0.4)$.
Also as $x \to \infty$, $y \to \infty$. And as $x \to -\infty$, $y \to 0$.



8 Marks: 1 for $f'(x)$, 1 for $f'(x) = 0 \Rightarrow x = 0, -2$, 1 for classifying $x = 0$ as yielding a local minimum, 1 for classifying $x = -2$ as yielding a local maximum, 1 for $f''(x)$, 1 for $f''(x) = 0 \Rightarrow x = -2 \pm \sqrt{2}$, 1 for the coordinates of the two points of inflection and 1 for the graph.

4. The number, N, of people who have heard a rumor spread by mass media at time, t, in days, is modelled by

$$N(t) = \frac{a}{1 + be^{-kt}}.$$

(a) If 50 people have heard the rumour initially and 300,000 people hear the rumour eventually, find a and b.

Solution: $N(0) = \frac{a}{1+b} = 50$ and as $t \to \infty$, $N \to 300000$, so $a = 300000$. And so $\frac{300000}{1+b} = 50$ or $b = 5999$.
4 Marks: 1 for $\frac{a}{1+b} = 50$, 1 for $t \to \infty$ implies $N \to a$, 1 for a and 1 for b.

(b) If the rumour is initially spreading at the rate of 500 people per day, find k.

Solution: $N(t) = \dfrac{300000}{1 + 5999e^{-kt}}$, so $N'(t) = \dfrac{300000 \times 5999 \times ke^{-kt}}{(1 + 5999e^{-kt})^2}$.
$N'(0) = \dfrac{300000 \times 5999 \times k}{(6000)^2} = 500$, so $k = \dfrac{60000}{5999} \approx 10$.
3 Marks: 1 for $N'(t)$, 1 for $N'(0)$ and 1 for $k \approx 10$.