The University of Sydney
School of Mathematics and Statistics

Tutorials for end of Week 10 / beginning of Week 11

MATH1111: Introduction to Calculus — Semester 1, 2011

Web Page: `http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/`
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1. Which of the following functions represents a plane?

(a) $z = 2x$
(b) $f(x, y) = 4(x - y)$
(c) $z = x(2y - 1)$
(d) $z = 4(x + y) - 3(x - y)$
(e) $f(x, y) = 4x - 2y^2 + 4$
(f) $f(x, y) = 4x(x + 1) + 2x(3 - 2x) + y$

2. Let

$$P = (1, 2, 3), \quad Q = (-1, 0, 1), \quad R(1, 0, 3)$$

be points in space. For each of the following planes, determine which (if any) of these points lie on the plane.

(a) $x + y + z = 6$
(b) $2x - 2y + z = 5$
(c) $-x + y = 1$
(d) $3x - 2y + 4z = -6$
(e) $3x - 3z = -6$

3. Find the distance d between the following pairs of points:

(a) $(0, 0, 0)$ and $(2, -4, 4)$.
(b) $(4, 6, 1)$ and $(-2, 0, 8)$.
(c) $(-1, 2, -9)$ and $(-2, 0, -7)$.

4. (a) Find the equation of the sphere of radius 5 centered at the origin.
(b) Find the equation of the sphere of radius 5 centered at $(1, 4, 2)$.
(c) Find the equation of the two planes that just touch the sphere in part (a) and are parallel to the xz-plane.

5. Find a parametric equation for a line passing through the point (1,4,5) that is parallel to (7,2,1).

6. Sketch graphs of the planes given by the following equations:

(a) $x = 3$
(b) $y = 1$
(c) $z = 2$
(d) $x + y + z = 1$

7. Suppose that z is a linear function of x and y with slope 3 in the x direction and slope 4 in the y direction.

(a) If $z = 0$ when $x = 5$ and $y = 2$, what is the value of z when $x = 6$ and $y = 4$?

(b) If $z = 2$ when $x = 3$ and $y = 3$, what is the value of z when $x = 2$ and $y = 2.5$?

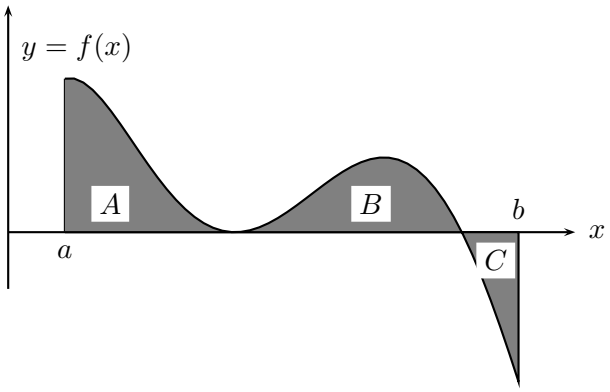
8. The speed v of a car is measured every two seconds for 10 seconds, and the results tabulated below:

t	0	2	4	6	8	10
$v(t)$	0	4	9	10	11	15

(a) Construct an upper and lower estimate for the distance the car has travelled during this time.

(b) How could you improve these estimates?

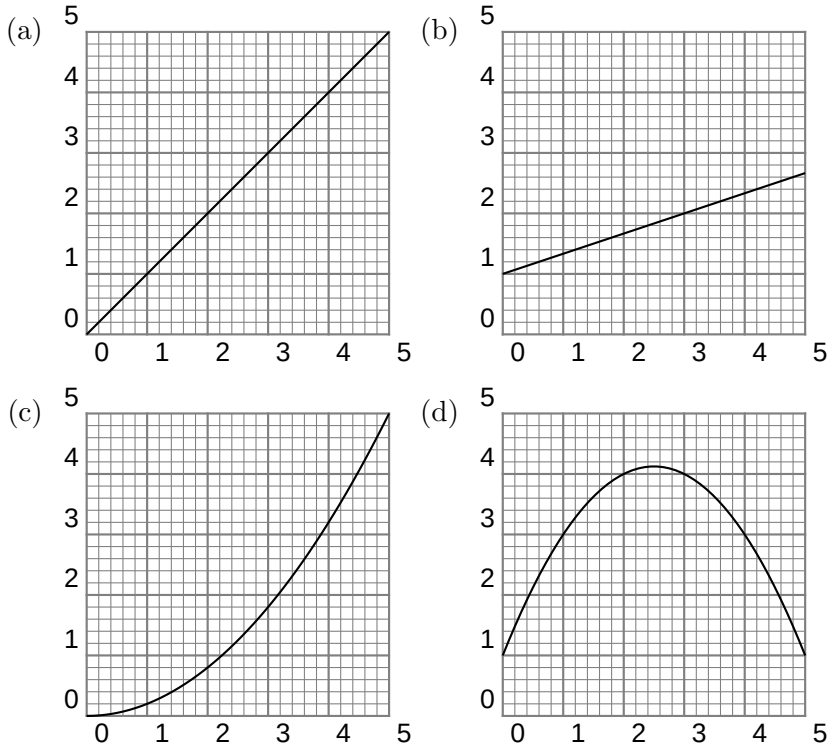
9. In the following diagram, the indicated areas A, B and C represent the area enclosed by the graph of the function $f(x)$, the x-axis and the lines $x = a$ and $x = b$.



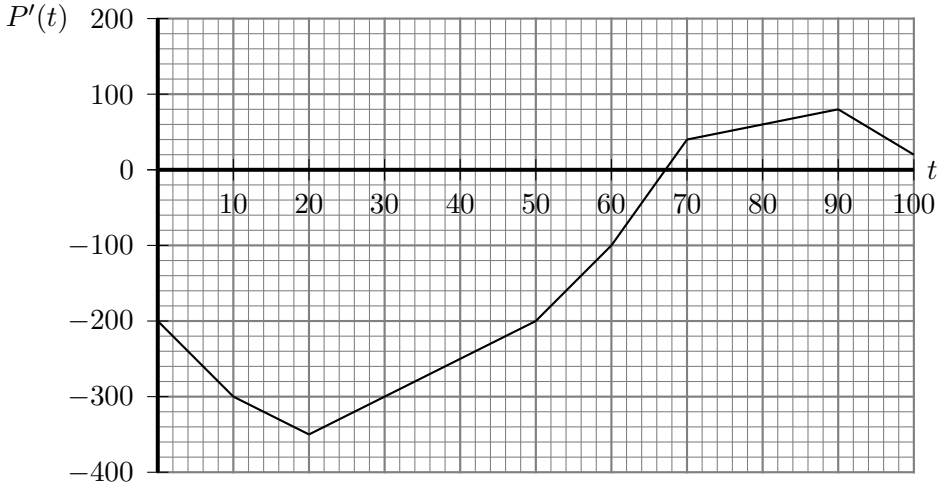
Express $\int_a^b f(x)\,dx$ in terms of A, B and C.

10. For each of the following functions, find an estimate of the integral of the function between

$x = 0$ and $x = 5$.



11. The graph below represents the rate of change of the population P of the whale species *Balaenoptera musculus*, where t is measured in years from 1910:



If the population of *Balaenoptera musculus* was 16 500 whales in 1910, find the population in:

(a) 1920,

(b) 1967,

(c) 2010.