The University of Sydney
School of Mathematics and Statistics

Tutorials for end of Week 3 / beginning of Week 4

MATH1111: Introduction to Calculus — Semester 1, 2011

Web Page: `http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/`
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1. Explain the following equations using logarithm laws:

(a) $\ln(\frac{1}{2}) = -\ln(2)$,
(b) $\ln(2e) = 1 + \ln(2)$,
(c) $\ln(100) = 2(\ln(5) + \ln(2))$.

2. Solve the following for x:

(a) $3^x = 17$,
(b) $20 = 50(1.04)^x$,
(c) $2^x = e^{x+1}$,
(d) $3e^{2x} = 5e^{4x}$.

3. Caffeine is eliminated from the body at a continuous rate of 17% per hour. A standard cup of coffee contains 150 mg of caffeine.

(a) Write a formula for the amount of caffeine, C, in mg as a function of t, the number of hours after drinking a cup of coffee.
(b) Use logarithms to find the number of hours it takes for half of the caffeine from a cup of coffee to be eliminated from the body.
(c) Use logarithms to find the number of hours it takes for only 1% of the caffeine from a cup of coffee to remain in your system.

4. For each of the following angles, determine which of the sine, cosine and tangent of the angle are positive, negative, zero or undefined.

(a) $\frac{3\pi}{4}$,
(b) $\frac{\pi}{4}$,
(c) 2π,
(d) $\frac{3\pi}{2}$,
(e) 4,
(f) -1.

5. Calculate the following sin and cos values without using your calculator.

(a) $\sin\left(-\frac{\pi}{6}\right)$ given that $\sin\left(\frac{\pi}{6}\right) = 0.5$,
(b) $\cos\left(\frac{\pi}{6}\right)$ given that $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$,
(c) $\sin\left(\frac{9\pi}{10}\right)$ given that $\cos\left(\frac{2\pi}{5}\right) = 0.309$.

6. For each of the following sinusoidal functions, determine its amplitude and period, and hence sketch its graph.

(a) $y = 4 + \cos(3x)$,

(b) $y = 2\sin(2x) + 1$,

(c) $y = 2 + 2\sin(\pi x)$.

7. Find a solution for x in each of the following equations, if possible:

(a) $1 = 6\cos(3x - 1) - 2$,

(b) $\sqrt{3} = 2\sin(5x)$,

(c) $-2 = \cos(0.5x + 1) + 1$.

8. The distance, d, that a thrown object travels is a function of its initial velocity, v, and the angle at which it is thrown, θ. The function is given by:

$$d = \frac{v^2 \sin(2\theta)}{g},$$

where g is the accelleration due to gravity.

(a) Sketch d as a function of θ for $0 \leq \theta \leq \pi/2$.

(b) What value of θ gives the largest possible value for d?

(c) What is this value of d?