1. Simplify the following:
   (a) \(6e^{\ln(B^2)}\),
   (b) \(\ln(2^{3AC})\),
   (c) \(2e^{\ln(2)+A}\),
   (d) \((e^{\ln(B^2)})^2\),
   (e) \(\ln(2e^{AB})\),
   (f) \(2\ln(e^B) + 3\ln(B^e)\).

   \textbf{Solution:}
   (a) \(6B^2\),
   (b) \(3AC\),
   (c) \(4e^A\),
   (d) \(B^4\),
   (e) \(\ln(2) + AB\).
   (f) \(2B + 3e\ln(B)\).

2. Explain the following equations using logarithm laws:
   (a) \(\ln\left(\frac{1}{2}\right) = -\ln(2)\),
   (b) \(\ln(2e) = 1 + \ln(2)\),
   (c) \(\ln(100) = 2(\ln(5) + \ln(2))\).

   \textbf{Solution:}
   (a) \(\ln\left(\frac{1}{2}\right) = \ln(2^{-1}) = -\ln(2)\),
   (b) \(\ln(2e) = \ln(2) + \ln(e) = 1 + \ln(2)\),
   (c) \(\ln(100) = \ln(25 \times 4) = \ln(25) + \ln(4) = \ln(5^2) + \ln(2^2) = 2\ln(5) + 2\ln(2) = 2(\ln(5) + \ln(2))\).

3. Solve the following for \(x\):
   (a) \(3^x = 17\),
   (b) \(20 = 50(1.04)^x\),
   (c) \(2^x = e^{x+1}\),
   (d) \(3e^{2x} = 5e^{4x}\).

   \textbf{Solution:}
   (a) \(x = 2.58\),
   (b) \(x = -23.4\),
(c) \( x = -3.26 \),
(d) \( x = -0.255 \).

4. Caffeine is eliminated from the body at a continuous rate of 17\% per hour. A standard cup of coffee contains 150 mg of caffeine.

(a) Write a formula of the form \( C = C_0(1 + r)^t \) for the amount of caffeine in mg, \( C \), as a function of the number of hours, \( t \), after drinking a cup of coffee.
(b) Find the number of hours it takes for half of the caffeine from a cup of coffee to be eliminated from the body.
(c) Find the number of hours it takes for only 1\% of the caffeine from a cup of coffee to remain in your system.

**Solution:**

(a) \( C = 150(1 - 0.17)^t = 150(0.83)^t \).
(b) Solving \( 0.5 = (0.83)^t \) we have \( \ln(0.5) = t \ln(0.83) \) or \( t = 3.72 \) hours.
(c) Solving \( 0.01 = (0.83)^t \) we have \( \ln(0.01) = t \ln(0.83) \) or \( t = 24.7 \) hours.

5. A sum of $5000 is invested at an interest rate of 5\% per year. Find the time required for the money to double if the interest is compounded:

(a) Annually.
(b) Continuously i.e. according to a formula of the form \( A = A_0e^{kt} \).

**Solution:**

(a) Using \( A = 5000(1 + 0.05)^t \), find \( t \) for which \( A = 10000 \). This gives \( 2 = 1.05^t \) and \( t = \frac{\ln(2)}{\ln(1.05)} \approx 14.21 \). The money will double in 14.21 years.
(b) Here have \( A = 5000e^{0.05t} \) and so \( 2 = e^{0.05t} \) giving \( t = \frac{\ln(2)}{0.05} \approx 13.86 \). The money will double in 13.86 years.

6. For each of the following angles, determine which of the sine, cosine and tangent of the angle are positive, negative, zero or undefined.

(a) \( \frac{5\pi}{4} \),
(b) \( \frac{2\pi}{3} \),
(c) \( 2\pi \),
(d) \( \frac{3\pi}{2} \),
(e) \( 4 \),
(f) \( -1 \).

**Solution:**

(a) \( \sin \) is positive, \( \cos \) is negative, \( \tan \) is negative.
(b) \( \sin \) is positive, \( \cos \) is positive, \( \tan \) is positive.
(c) \( \sin \) is zero, \( \cos \) is positive, \( \tan \) is zero.
(d) \( \sin \) is negative, \( \cos \) is zero, \( \tan \) is undefined.
(e) \( \sin \) is negative, \( \cos \) is negative, \( \tan \) is positive.
(f) sin is negative, cos is positive, tan is negative.

7. Calculate the following sin and cos values without using your calculator, but using your table of Special Values.

(a) \( \sin \left( -\frac{\pi}{6} \right) \) given that \( \sin \left( \frac{\pi}{6} \right) = 0.5 \)

(b) \( \cos \left( \frac{\pi}{6} \right) \) given that \( \cos \left( -\frac{\pi}{6} \right) = \frac{\sqrt{3}}{2} \)

Solution:

(a) \( \sin \left( -\frac{\pi}{6} \right) = -\sin \left( \frac{\pi}{6} \right) = -0.5 \)

(b) \( \cos \left( \frac{\pi}{6} \right) = \cos \left( -\frac{\pi}{6} \right) = \frac{\sqrt{3}}{2} \)

8. For each of the following sinusoidal functions, determine its amplitude and period, and hence sketch its graph.

(a) \( y = 4 + \cos(3x) \),

(b) \( y = 2 \sin(2x) + 1 \),

(c) \( y = 2 + 2 \sin(\pi x) \),

(d) \( y = 3 \sin(2x - \frac{\pi}{2}) \).

Solution:

(a) ![Graph of y = 4 + cos(3x)](image)

Amplitude: 1. Period: \( \frac{2\pi}{3} \).

(b) ![Graph of y = 2sin(2x) + 1](image)

Amplitude: 2. Period: \( \pi \).
9. Find all solutions $x$ for the following equations, where $-\pi \leq x \leq \pi$.

(a) $\sin(2x) = \frac{1}{2}$,
(b) $\sin(x) = -\frac{1}{2}$,
(c) $\sqrt{3} = 2\sin(5x)$,
(d) $1 = 6\cos(3x - \pi) - 2$,
(e) $-2 = \cos(0.5x + 1) + 1$.

Solution:

(a) $x = \frac{\pi}{12}, \frac{5\pi}{12}$,
(b) $x = -\frac{\pi}{6}$,
(c) $x = -\frac{5\pi}{15}, -\frac{4\pi}{15}, -\frac{\pi}{15}, -\frac{2\pi}{15}, -\frac{7\pi}{15}$,
(d) $x = -\frac{4\pi}{9}, -\frac{2\pi}{9}, -\frac{2\pi}{9}, -\frac{4\pi}{9}$,
(e) No solution exists.
10. As a wave passes by an offshore piling, the height of the water is modelled by the function

\[ h(t) = 3 \cos \left( \frac{\pi}{10} t \right) \]

where \( h(t) \) is the height in metres above the mean sea level at time \( t \) seconds.

(a) Find the period of the wave.
(b) Find the vertical distance between the trough and the crest of the wave.
(c) What will be the height of the wave after 1 minute?
(d) Give a time when the height of the wave is \(-1\) metres.

**Solution:**

(a) 20.
(b) 6.
(c) \( h(60) = 3 \) metres.
(d) \( h(t) = -1 \) gives \( t = 6.08 \) seconds.

11. The distance, \( d \), that a thrown object travels is a function of its initial velocity, \( v \), and the angle at which it is thrown, \( \theta \). The function is given by:

\[ d = \frac{v^2 \sin(2\theta)}{g}, \]

where \( g \) is the acceleration due to gravity.

(a) Sketch \( d \) as a function of \( \theta \) for \( 0 \leq \theta \leq \pi/2 \).
(b) What value of \( \theta \) gives the largest possible value for \( d \)?
(c) What is this value of \( d \)?

**Solution:**

(a)

\[ d \]

\[ \frac{v^2}{g} \]

\[ \frac{\pi}{8} \]

\[ \frac{2\pi}{8} \]

\[ \frac{3\pi}{8} \]

\[ \frac{4\pi}{8} \]

\[ \theta \]

(b) \( \frac{\pi}{4} \).
(c) \( \frac{v^2}{g} \).