1. For each of the functions below, determine which function dominates as $x \to \infty$:

(a) $f(x) = 0.1 \times 2^x$ and $g(x) = 100x^{20}$,
(b) $f(x) = \ln(2^x)$ and $g(x) = \ln(x^2)$.
(c) $f(x) = \ln(x)$ and $g(x) = \sqrt{x}$.

**Solution:**

(a) $f(x)$,
(b) $f(x)$,
(c) $g(x)$.

2. For each of the graphs below, determine the minimal possible degree of the polynomial, and whether the leading coefficient of the polynomial is positive or negative:

![Graphs](image)

**Solution:**

(a) Minimum degree = 3, leading coefficient negative.
(b) Minimum degree = 4, leading coefficient posative.
(c) Minimum degree = 5, leading coefficient negative.
(d) Minimum degree = 3, leading coefficient positive.

3. For each of the functions below, describe what happens to $f(x)$ as $x \to \pm\infty$:

(a) $f(x) = e^x$,
(b) $f(x) = x^2 + x + 1$,
(c) $f(x) = 12 + 3x^2 + 4x^3 - 3x^4$,
(d) $f(x) = \frac{2x^2 - 13x + 4}{3x^2 + 1}$,
(e) $f(x) = \frac{x^3 + 3}{3x^3 - 1}$.

**Solution:**

(a) $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to 0$ as $x \to -\infty$.
(b) $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to \infty$ as $x \to -\infty$.
(c) $f(x) \to -\infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$.
(d) $f(x) \to \frac{2}{3}$ as $x \to \infty$ and $f(x) \to \frac{2}{3}$ as $x \to -\infty$,
(e) $f(x) \to \frac{1}{3}$ as $x \to \infty$ and $f(x) \to \frac{1}{3}$ as $x \to -\infty$.

4. Find possible formulae for each of the polynomial functions below:

(a) A possible solution is $f(x) = x(x + 1)(2 - x) = -x^3 + x^2 + 2x$. 

**Solution:**

(a) A possible solution is $f(x) = x(x + 1)(2 - x) = -x^3 + x^2 + 2x$. 

(2)
(b) A possible solution is \( f(x) = (x + 3)(x - 1)(x - 2) = x^3 - 7x + 6 \).
(c) A possible solution is \( f(x) = (x + 3)(x - 1)^2(3 - x) = -x^4 + 2x^3 + 8x^2 - 18x + 9 \).
(d) A possible solution is \( f(x) = x^2(2 - x)(x + 2) + 1 = -x^4 + 4x^2 + 1 \).

5. For each of the following functions,

\[ g(x) = \frac{x^2 - 1}{x^2 + 1}, \quad h(x) = \frac{x^2 + 1}{x^2 - 1}. \]

Decide which of the following descriptions apply:

(a) The function has a horizontal asymptote at \( y = 1 \),
(b) The function is symmetrical about the \( y \)-axis,
(c) The function is odd,
(d) The function has vertical asymptotes at \( x = \pm 1 \).

**Solution:**

(a) This applies to the functions \( g(x) \) and \( h(x) \).
(b) This applies to the functions \( g(x) \) and \( h(x) \).
(c) This applies to none of the functions.
(d) This applies to the function \( h(x) \).