

Tutorials for end of Week 7 / beginning of Week 8

MATH1111: Introduction to Calculus

Semester 1, 2011

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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1. Differentiate the following functions:

- (a) $y = (x + 1)^{100}$
- (b) $f(x) = (x^2 + x)^4$
- (c) $g(t) = \sqrt{1 + t^3}$
- (d) $h(x) = \frac{1}{(2x^2 + x + 1)^3}$
- (e) $f(t) = (t^3 + \sqrt{t})^{\frac{3}{2}}$

2. Differentiate the following functions:

- (a) $y = e^{3t}$
- (b) $f(x) = e^{x^2}$
- (c) $g(x) = e^{\sqrt{x}+1}$
- (d) $y = \sqrt{e^x + x^3}$
- (e) $f(t) = (t^2 + 2e^{2t})^4$

3. Differentiate the following functions:

- (a) $f(x) = \left(\frac{x}{1+x^2}\right)^3$
- (b) $y = t^2 e^{5-t^2}$
- (c) $y = e^{\frac{t}{1+t}}$
- (d) $h(x) = e^{e^{x^2}}$
- (e) $g(x) = e^{(1+x)^{50}}$

4. (a) Show that $y(t) = Ce^{kt}$ is a solution to the differential equation

$$\frac{dy}{dt} = ky.$$

- (b) Find the solution to the differential equation $dy/dt = -5y$ with the initial value $y(0) = 10$.
- (c) Find the solution to the differential equation $dy/dt = 11y$ where $y(2) = 5$.
- (d) Find the solution to the differential equation

$$2\frac{dy}{dt} + 4y = 0$$

where $y(1) + y(2) = 12$.

5. The theory of special relativity establishes a connection between an object's velocity, v , and its mass, m , given by the following formula:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where m_0 is the object's mass when it has zero velocity (rest mass), and c is the speed of light.

- (a) Suppose an object with a 60kg rest mass is accelerated to 99% of the speed of light. What is its mass at this speed?
- (b) At what velocity does the object weigh twice as much as its rest mass? Give your answer as a percentage of c .
- (c) Find $\frac{dm}{dv}$.
6. For what values of x is the function $n(x) = e^{-x^2}$ concave down?
7. Let $f(x) = (3x + 1)^3(2x - 4)^4$ and $g(x) = (3x + 1)^2(2x - 4)^3$.
- (a) Find $f'(x)$.
- (b) Find $g'(x)$.
- (c) Using part (b) and appropriate factorisation, or otherwise, find $f''(x)$.
8. Differentiate the following functions:
- (a) $f(x) = \ln(x^2)$
- (b) $g(t) = \ln(2t)$
- (c) $y = \ln(e^{4x^3+x^2-4})$
- (d) $h(x) = x \ln x - xe^x + x^3 - 1$
- (e) $y = \frac{x}{1+\ln x}$
- (f) $g(x) = x^x$
- (g) $P(t) = P_0(1 + r)^t$
9. Find the equation of the tangent line of each of the following functions at the given x value:
- (a) $f(x) = \ln x$ at $x = 2$
- (b) $g(x) = 2x \ln x$ at $x = 3$
- (c) $h(x) = e^{x^3}$ at $x = 1$