

**Tutorials for end of Week 8 / beginning of Week 9**

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MATH1111: Introduction to Calculus

Semester 1, 2011

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Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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1. Find the first derivatives of the following functions:

(a)  $y = \sin \theta + 2 \cos \theta$

(b)  $f(\theta) = \cos(5\theta)$

(c)  $g(x) = \sin\left(\frac{\theta}{2}\right)$

(d)  $y = \cos x \sin x$

(e)  $f(x) = \tan x \cos x$

(f)  $h(\theta) = \sqrt{1 - \sin \theta}$

2. Find the first derivatives of the following functions:

(a)  $y = e^{\cos t}$

(b)  $y = \cos(e^t)$

(c)  $f(\theta) = \cos(\sin \theta)$

(d)  $h(\theta) = \sin^6 \theta$

(e)  $g(x) = \frac{x}{\cos x}$

(f)  $f(x) = \frac{1 - \cos^2 x}{1 - \sin^2 x}$

3. Find the first and second derivatives of the following functions:

(a)  $g(t) = \sin(\pi t)$

(b)  $f(\theta) = \theta^2 \cos \theta$

(c)  $h(x) = \cos^2 x + 2 \cos x + 4$

(d)  $y = \sin^2 t - \sin(t^2)$

(e)  $f(\theta) = \tan \theta$

4. (a) Write down the first five derivatives of  $y = \sin 2x$ .

(b) Find the 10<sup>th</sup> derivative of  $y$ .

5. Australian homes and businesses use Alternating Current with peak voltage 220 Volts and frequency 50 Hertz to power electrical appliances. The voltage of an outlet is given as a function of  $t$ , measured in seconds:

$$V = V_{\text{peak}} \cos(2\pi ft),$$

where  $V_{\text{peak}}$  is the peak voltage, and  $f$  is the frequency of the current.

(a) Write an expression for the voltage of a standard Australian outlet.

(b) How many seconds does it take for the voltage to drop from peak voltage to minimum voltage and return to peak voltage?

(c) Find the rate of change of voltage of a standard Australian outlet.

(d) What is the maximum value of this rate of change?

6. Suppose a function  $f$  has continuous derivative, and the value of its derivative at certain values of  $x$  are given in the table below:

$x$	0	1	2	3	4	5	6	7	8	9	10
$f'(x)$	4	2	0	2	1	-1	-3	-1	2	5	10

- (a) Estimate the values for  $x$  where the function  $f$  has a critical point.
- (b) For each of the points you found in part (a), determine whether the point is a local maximum, a local minimum, or neither.
7. For each of the functions below, find and classify the critical points as local maxima, local minima, or neither.
- (a)  $y = x^3 - 2x^2 + x$
- (b)  $y = x^4 - 3x^3 + x^2 + 1$
- (c)  $f(x) = x + x^{-2}$ ,  $x > 0$
- (d)  $g(x) = 10x^2e^{-2x}$
- (e)  $h(\theta) = \theta \ln \theta$ ,  $\theta > 0$
- (f)  $y = e^t - 2t$
8. For the functions in parts (a) and (b) in the previous question, find any points of inflection.
9. For each of the following polynomials,
- (i) Find all critical points,
- (ii) Classify each critical point as a local maximum, local minimum, or neither,
- (iii) Find all points of inflection,
- (iv) Find the corresponding  $y$ -values for each of the points you have found in parts (i)-(iii), and hence graph the polynomial.
- (a)  $y = x^2 + 3x + 2$
- (b)  $y = x^3 - 9x^2 + 24x + 3$
- (c)  $y = x^4 - 2x^3 + x^2 + 1$