

Tutorials for end of Week 9 / beginning of Week 10

MATH1111: Introduction to Calculus

Semester 1, 2011

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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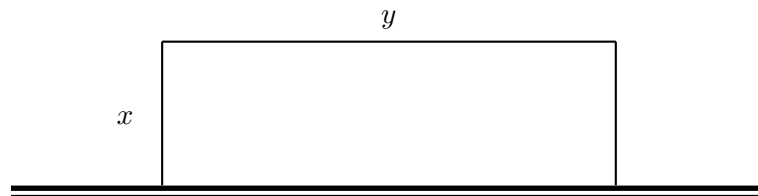
- Let $F(x) = 2x^3 - 9x^2 + 12x + 4$.
 - Find all critical points of $F(x)$.
 - Classify the critical points of $F(x)$ as local maxima, local minima or neither.
 - What are the maximum and minimum values of $F(x)$ on the interval $-1.5 \leq x \leq 1.5$?
- For each of the following functions, find the global maximum and global minimum on the given interval.
 - $y = -x^2 - 3x + 3$ for $-4 \leq x \leq 0$.
 - $f(\theta) = \theta + \sin \theta$ for $1 \leq \theta \leq 4$.
 - $g(t) = t \ln t$ for $2 \leq t \leq 3$.
 - $H(x) = xe^{-x}$ for $0 \leq x \leq 5$.

- The population of termites in a termite mound is modelled by the equation

$$f(t) = 10000 - \frac{9900}{1+t},$$

where t is measured in days.

- What is the initial population of the termites?
 - How many days will it take for the population to reach 9000?
 - Find $f'(t)$ and $f''(t)$.
 - What happens to the growth rate of the population as $t \rightarrow \infty$? What happens to the population?
- A principal at a primary school wants to fence off an area of a large playground for a vegetable garden, using an existing wall as one side:



She has 20m of fencing available.

- What are the values of x and y that maximize the area of the garden?
- What is the maximum area of the garden?

5. The efficiency of a screw, E , is given by

$$E = \frac{\theta - \mu\theta^2}{\mu + \theta},$$

where $\theta > 0$ is the angle of pitch of the thread, and $\mu > 0$ is the coefficient of friction of the material. What value of θ maximises E ?

6. For some positive constant C , a patient's temperature change, T , after being given a D milligram dose of a particular drug, is given by

$$T = \left(\frac{C}{2} - \frac{D}{3}\right) D^2.$$

- (a) What dosage maximises this temperature change?
(b) The sensitivity of the patient to the drug is defined as $\frac{dT}{dD}$. What dosage maximises sensitivity?
7. A boy pulls a billy cart which, together with its load, weighs m kg. The force he exerts in pulling this cart varies with the angle, θ , that his arm makes with his body. The least force he must exert to move the cart is modelled by the equation

$$F = \frac{mg\mu}{\sin\theta + \mu\cos\theta},$$

where μ is the coefficient of friction, and g is the force due to gravity. If $\mu = 0.2$ and $g = 9.8$, find the maximum and minimum values of F for $0 \leq \theta \leq \frac{\pi}{2}$ (give your answer in terms of m).

8. A small furniture business signs a contract with a customer to deliver up to 400 chairs on a regular basis, with the exact number varying from week to week. The price is set at \$90 per chair for the first 300 chairs, and every chair purchased above 300 reduces the price of every chair in the order by \$0.25.
- (a) Suppose the order is between 0 and 300 chairs. What is the minimum and maximum amount the customer must pay for their order?
(b) Now suppose the order is between 300 and 400 chairs. Write an equation expressing the cost, C , of an order of n chairs as a function of n .
(c) What is the minimum and maximum amount the customer must pay for their order if they order between 300 and 400 chairs?
(d) Has the business signed a bad contract? Why or why not?

9. An electric current, I , measured in amps, is given by

$$I = \cos(\omega t) + \sqrt{5} \sin(\omega t),$$

where $\omega \neq 0$ is a constant. What are the maximum and minimum values of I ?

Most questions in this tutorial are drawn from "Calculus: Single and Multivariable", by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum et al., John Wiley & Sons, 4th ed.