1. Let \( F(x) = 2x^3 - 9x^2 + 12x + 4 \).
   (a) Find all critical points of \( F(x) \).
   (b) Classify the critical points of \( F(x) \) as local maxima, local minima or neither.
   (c) What are the maximum and minimum values of \( F(x) \) on the interval \(-1.5 \leq x \leq 1.5\)?

2. For each of the following functions, find the global maximum and global minimum on the given interval.
   (a) \( y = -x^2 - 3x + 3 \) for \(-4 \leq x \leq 0\).
   (b) \( f(\theta) = \theta + \sin \theta \) for \(1 \leq \theta \leq 4\).
   (c) \( g(t) = t \ln t \) for \(2 \leq t \leq 3\).
   (d) \( H(x) = xe^{-x} \) for \(0 \leq x \leq 5\).

3. The population of termites in a termite mound is modelled by the equation
   \[
   f(t) = 10000 - \frac{9900}{1 + t},
   \]
   where \( t \) is measured in days.
   (a) What is the initial population of the termites?
   (b) How many days will it take for the population to reach 9000?
   (c) Find \( f'(t) \) and \( f''(t) \).
   (d) What happens to the growth rate of the population as \( t \to \infty \)? What happens to the population?

4. A principal at a primary school wants to fence off an area of a large playground for a vegetable garden, using an existing wall as one side:

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     y
   __________
   |        |
   |        |
   x
   __________
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She has 20m of fencing available.
   (a) What are the values of \( x \) and \( y \) that maximize the area of the garden?
   (b) What is the maximum area of the garden?

5. An open rectangular box with square base is to be made from 48 cm\(^2\) of material. What dimensions will result in a box with the largest possible volume?
6. A television manufacturing firm needs to design an open-topped box with a square base. The box must hold 32 cm³. Find the dimensions of the box that can be built with the minimum amount of materials.

7. A sheet of metal 3m by 4m will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with the largest volume?

8. A fence must be built in a large field to enclose a rectangular area of 15,625m². One side of the area is bounded by an existing fence; no fence is needed there. Material for the fence costs $2 per metre for the two ends, and $4 per metre for the side opposite the existing fence. Find the cost, $C$, of the least expensive fence.

9. A container in the shape of a circular cylinder with no top has surface area $3\pi$ cm². What height, $h$, and base radius, $r$, will maximise the volume, $V$, of the cylinder?

10. A young man is at a point on a river bank. He wants to get to his cabin that is on the other side of the river and is 8km west and 3km north of where he currently stands. The river runs east to west. He can travel at 4km/hr on the river but jog at 5km/hr on the land. How far up the river bank should he go before crossing in order to reach his cabin in minimum time?

11. A small furniture business signs a contract with a customer to deliver up to 400 chairs on a regular basis, with the exact number varying from week to week. The price is set at $90 per chair for the first 300 chairs, and every chair purchased above 300 reduces the price of every chair in the order by $0.25.

(a) Suppose the order is between 0 and 300 chairs. What is the minimum and maximum amount the customer must pay for their order?

(b) Now suppose the order is between 300 and 400 chairs. Write an equation expressing the cost, $C$, of an order of $n$ chairs as a function of $n$.

(c) What is the minimum and maximum amount the customer must pay for their order if they order between 300 and 400 chairs?

(d) Has the business signed a bad contract? Why or why not?

12. (*) The efficiency of a screw, $E$, is given by

$$E = \frac{\theta - \mu \theta^2}{\mu + \theta},$$

where $\theta > 0$ is the angle of pitch of the thread, and $\mu > 0$ is the coefficient of friction of the material. What value of $\theta$ maximises $E$?

13. (*) For some positive constant $C$, a patient’s temperature change, $T$, after being given a $D$ milligram dose of a particular drug, is given by

$$T = \left( \frac{C}{2} - \frac{D}{3} \right) D^2.$$ 

(a) What dosage maximises this temperature change?

(b) The sensitivity of the patient to the drug is defined as $\frac{dT}{dD}$. What dosage maximises sensitivity?
14. (*) A boy pulls a billy cart which, together with its load, weighs $m$ kg. The force he exerts in pulling this cart varies with the angle, $\theta$, that his arm makes with his body. The least force he must exert to move the cart is modelled by the equation

$$F = \frac{mg\mu}{\sin \theta + \mu \cos \theta},$$

where $\mu$ is the coefficient of friction, and $g$ is the force due to gravity. If $\mu = 0.2$ and $g = 9.8$, find the maximum and minimum values of $F$ for $0 \leq \theta \leq \frac{\pi}{2}$ (give your answer in terms of $m$).