

Solutions to Tutorials 06/03/08,07/03/08 and 10/03/08,11/03/08

MATH1111: Introduction to Calculus

Semester 1, 2008

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 1.1: 1, 4, 5, 6, 7, 10.

Solution:

1. $f(35)$ means the value of P corresponding to $t = 35$. Since t represents the number of years since 1950, $f(35)$ means the population of the city in 1985. So, in 1985, the city's population was 12 million.

4. The slope is $\frac{1-0}{1-0} = 1$, and so the equation of the line is $y = x$.

5. The slope is $\frac{3-2}{2-0} = \frac{1}{2}$, and so the equation of the line is $y = \frac{1}{2}x + 2$.

6. The slope is $\frac{3-1}{2+2} = \frac{1}{2}$, and so the equation of the line is $y - 1 = \frac{1}{2}(x + 2)$ or $y = \frac{1}{2}x + 2$.

7. $y = -\frac{12}{7}x + \frac{2}{7}$, so the slope is $-\frac{12}{7}$ and the y -intercept is $(0, \frac{2}{7})$.

10. (a) V, (b) IV, (c) I, (d) VI, (e) II, (f) III.

2. Find the equation of the line that passes through the points $(2, 3)$ and $(2, -6)$.

Solution: If we calculate the slope of the line, we find $\frac{-6-3}{2-2} = \frac{-9}{0}$ which is undefined. Since both ordered pairs have x -coordinate 2, the equation of the line is $x = 2$. You could also deduce the equation $x = 2$ by first plotting the given ordered pairs and then drawing a line through them to find that we have a vertical line whose points all have x -coordinate 2.

3. Section 1.1: 31.

Solution:

(a) $(100, 212)$ and $(0, 32)$ are two points on the line so the slope is $\frac{212-32}{100-0} = \frac{9}{5}$.

(b) $F - 32 = \frac{9}{5}C$ or $F = 1.8C + 32$.

(c) When $C = 20$, $F = 1.8 \times 20 + 32 = 68$.

(d) When does $F = C$? When $F = 1.8F + 32$, or $F = -40$. So the temperature is -40° .

4. Section 1.1: 16, 17, 18, 19, 20.

Solution:

16. Domain: $-2 \leq x \leq 2$; Range: $-2 \leq y \leq 2$.

17. Domain: $1 \leq x \leq 5$; Range: $1 \leq y \leq 6$.

18. Domain: $(-\infty, \infty)$; Range: $[2, \infty)$.

19. Domain: $(-\infty, \infty)$; Range: $(0, \frac{1}{2}]$.

20. $f(t)$ is a real number provided $t^2 - 16 \geq 0$ or $t^2 \geq 16$. This occurs when either $t \geq 4$ or $t \leq -4$.

$f(t) = 3 \Rightarrow \sqrt{t^2 - 16} = 3$ or $t^2 - 16 = 9$ or $t = \pm 5$.

5. Section 1.2: 3, 4, 11, 12, 22, 26, 28, 31.

Solution:

3. Initial quantity = 3.2; growth rate = 3%.

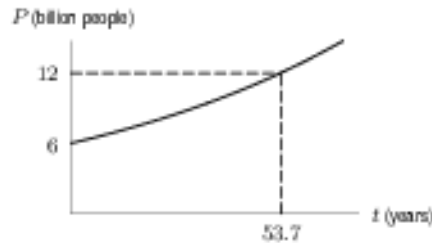
4. Initial quantity = 15; decay rate = 6%.

11. The function is increasing and concave up on the x -interval between D and E , and the x -interval between H and I . It is increasing and concave down on the x -interval between A and B , and the x -interval between E and F . It is decreasing and concave up on the x -interval between C and D , and the x -interval between G and H . Finally, it is decreasing and concave down on the x -interval between B and C , and the x -interval between F and G .

12. (a) Let P represent the world's population and t represent the number of years since 1999. Then $P = P_0(1.013)^t$ with $P_0 = 6$.

(b) 2020 corresponds to $t = 21$, so $P = 6(1.013)^{21} \approx 7.9$ billion people.

(c) The population of the world has doubled when $P = 12$. From the graph below we see that this occurs approximately when $t = 53.7$. The doubling time of the population of the world is about 53.7 years.



22. $f(0.02) = 25.02$ gives

$$Q_0 a^{0.02} = 25.02 \tag{1}$$

and $f(0.05) = 25.06$ gives

$$Q_0 a^{0.05} = 25.06. \tag{2}$$

Solving (1) and (2) simultaneously to eliminate Q_0 gives $a^{0.05-0.02} = 25.06/25.02$ or $a = 1.05$. And so $r = 0.05$.

26. $P = 7e^{-\pi t} = 7(e^{-\pi})^t = 7(0.0432)^t$. This represents exponential decay as $-\pi < 0$ or $0.0432 < 1$.

28. Say at 0 feet have air pressure P_0 . Then since every 100 feet air pressure decays by 0.4%, we have that at x hundred feet the air pressure P is 99.6% what it was 100 feet below or that $P = P_0(0.996)^x$. So at Mexico City (at 73.4 hundred feet) the air pressure is $P = P_0(0.996)^{73.4} = 0.75P_0$. So the air pressure is reduced 25%.

31. Let P_0 be the initial amount of strontium-90 absorbed in 1960, and P be the amount left after t years. Then can consider $P = P_0 e^{-kt}$. When $t = 29$ have $P = \frac{1}{2}P_0$. So $\frac{1}{2}P_0 = P_0 e^{-29k}$ giving $k = 0.0239$. So have $P = P_0 e^{-0.0239t}$. So in 1990 when $t = 30$, $P = P_0 e^{-0.0239 \times 30} = 0.488P_0$. The fraction remaining is 48.8%.