THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Solutions to Tutorials Weeks 11 and 12

MATH1111: Introduction to Calculus — Semester 1, 2009

Web Page: `http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/`
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Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 12.1: 2, 7, 8, 9, 10, 11, 18.

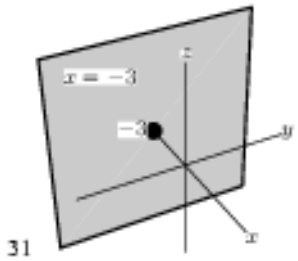
Solution:

2. A is closest to the yz-plane since it has the smallest x-coordinate in absolute value. B lies on the xz-plane since its y-coordinate is 0. C is farthest from the xy-plane since it has the largest z-coordinate in absolute value.

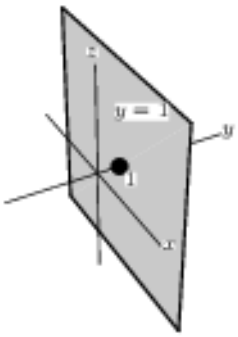
7. The equation is $x^2 + y^2 + z^2 = 25$.

8. The equation is $(x-1)^2 + (y-2)^2 + (z-3)^2 = 25$.

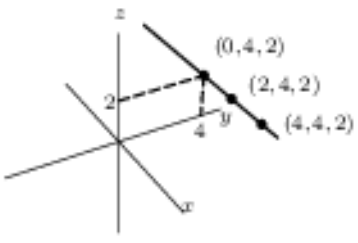
9. The graph is a plane parallel to the yz-plane and passing through the point $(-3, 0, 0)$.



10. The graph is a plane parallel to the xz-plane and passing through the point $(0, 1, 0)$.



11. The graph is all points with $y = 4$ and $z = 2$, i.e. a line parallel to the x-axis ad passing through the points $(0, 4, 2)$, $(2, 4, 4)$, $(4, 4, 2)$, etc.



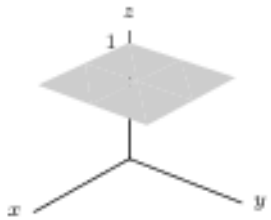
18. The gravitational force on a 100 kg object which is 7,000,000 meters from the centre of the earth is about 820 newtons.

2. Section 12.2: 1, 3, 4, 5, 6, 8, 9, 10.

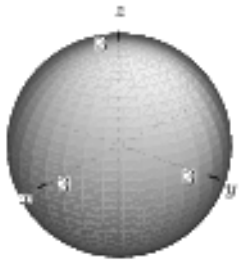
Solution:

1. (a) (IV) as $z = 2 + x^2 + y^2$ is a paraboloid opening upward with a positive z-intercept.
(b) (II) as $z = 2 - x^2 - y^2$ is a paraboloid opening downward.
(c) (I) as $z = 2(x^2 + y^2)$ is a paraboloid opening upward and going through the origin.
(d) (V) as $z = 2 + 2x - y$ is a slanted plane.
(e) (III) as $z = 2$ is a horizontal plane.

3. The graph is a horizontal plane 3 units above the xy-plane.

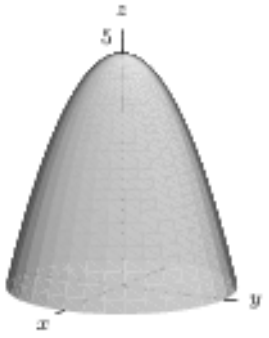


4. The graph is a sphere of radius 3, centered at the origin.



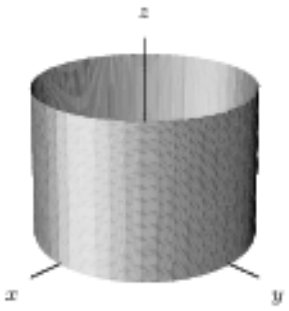
5. The graph is a bowl opening up, with vertex at the point $(0, 0, 4)$.

6. The graph is an upside-down bowl moved up 5 units and with vertex at $(0, 0, 5)$.

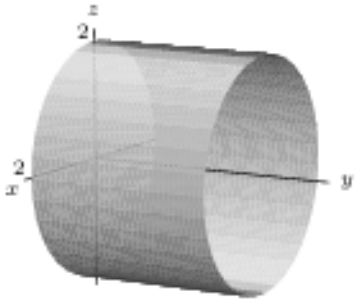


8. The graph is a plane with x-intercept 6, and y-intercept 3, and z-intercept 4.

9. In the xy-plane, the graph is a circle of radius 2. Since there are no restrictions on z, we extend this circle along the z-axis. The graph is a circular cylinder extended in the z-direction.



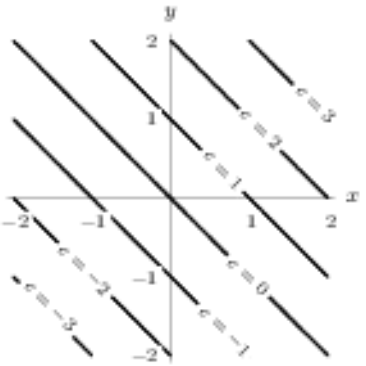
10. In the xz-plane, the graph is a circle of radius 2. Since there are no restrictions on y, we extend this circle along the y-axis. The graph is a circular cylinder extended in the y-direction.



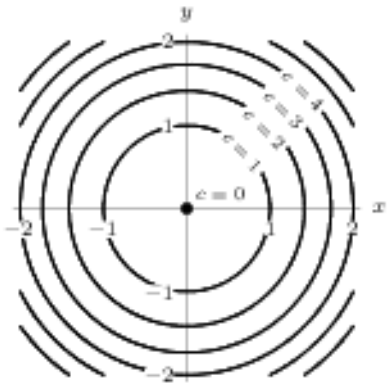
3. Section 12.3: 1, 3, 4, 17.

Solution:

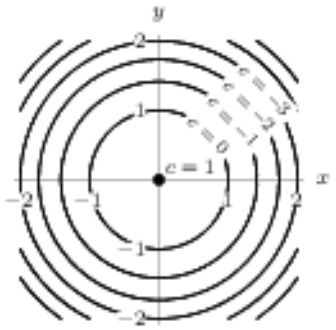
1. The contour where $f(x, y) = x + y = c$, or $y = -x + c$, is the graph of the straight line with slope -1. The contours are spaced evenly.



3. The contour where $f(x, y) = x^2 + y^2 = c$, where $c \geq 0$, is the graph of the circle centered at $(0, 0)$ with radius $\sqrt{c}$. The contours become more closely packed as we move further from the origin.



4. The contour where $f(x,y) = -x^2 - y^2 + 1 = c$, where $c \leq 1$, is the graph of the circle centered at $(0,0)$ with radius $\sqrt{1-c}$. The contours become more closely packed as we move further from the origin.



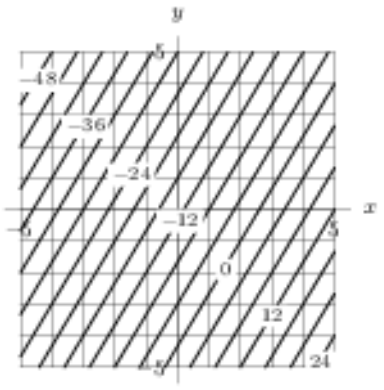
17. (a) (III), (b) (I), (c) (V), (d) (II), (e) (IV).

4. Section 12.4: 1, 3 (For part (b) find the contour for $z = 0$), 12, 13, 20, 22.

Solution:

1. (a) yes.
(b) The coefficient of m is 15 dollars per month. It represents the monthly charge to use this service. The coefficient of t is 0.05 dollars per minute. Each minute the customer is on-line costs 5 cents.
(c) The intercept represents the base charge. It costs \$35 to get hooked up to this service.
(d) $f(3,800) = 120$. A customer who uses this service for three months and is on-line for a total of 800 minutes is charged \$120.

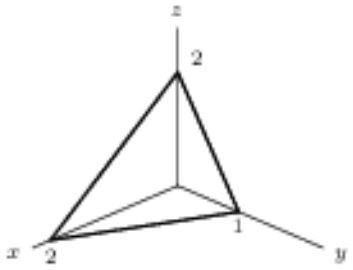
3. (a) Substituting in the values for the slopes, we see that the formula for the plane is $z = c + 5x - 3y$ for some value of c. Substituting the point $(4,3,-2)$ yields $c = -13$. The formula for the plane is $z = -13 + 5x - 3y$.
(b) When $z = 0$, we have $0 = -13 + 5x - 3y$ or $y = \frac{5}{3}x - \frac{13}{3}$. The contour for $z = 0$ is a line with slope 5/3 and y-intercept 13/3.



12. A contour diagram is linear if the contours are parallel straight lines, equally spaced for equally spaced values of z. This contour diagram does not represent a linear function.

13. A contour diagram is linear if the contours are parallel straight lines, equally spaced for equally spaced values of z. This contour diagram could represent a linear function.

20.



22.