

**Solutions to Tutorials 16/03/09,17/03/09 and 19/03/09,20/03/09**

MATH1111: Introduction to Calculus

Semester 1, 2009

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

Lecturer: Clio Cresswell

Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 1.4: 3, 4, 6, 9, 10, 11, 43 (a) (b) (c).

**Solution:**

3.  $5A^2$ .

4.  $2AB$ .

6.  $2A \ln(e) + 3e \ln(B) = 2A + 3e \ln(B)$ .

9.  $\frac{20}{50} = (1.04)^x$ ,  $\ln(2/5) = x \ln(1.04)$ ,  $x = \frac{\ln(0.4)}{\ln(1.04)}$ .

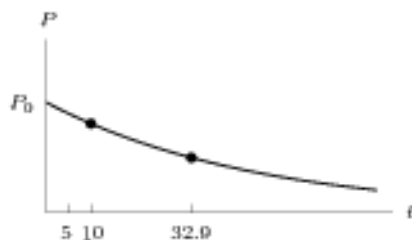
10.  $\ln(4) + x \ln(3) = \ln(7) + x \ln(5)$ ,  $x(\ln(3) - \ln(5)) = \ln(7) - \ln(4)$ ,  $x = \frac{\ln(7/4)}{\ln(3/5)}$ .

11.  $\ln(2^x) = \ln(e^{x+1})$ ,  $x \ln(2) = (x + 1) \ln(e)$ ,  $x \ln(2) = x + 1$ ,  $x(\ln(2) - 1) = 1$ ,  $x = \frac{1}{\ln(2)-1}$ .

43. (a) The decay follows the equation  $P = P_0 e^{-5k}$ , and 10% of the pollution is removed after 5 hours (meaning 90% is left). Therefore,  $0.90P_0 = P_0 e^{-5k}$  or  $k = -0.2 \ln(0.90)$ . Thus, after 10 hours  $P = P_0 e^{-10 \times (-0.2 \ln(0.90))}$  or  $P = 0.81P_0$ . So, 81% of the original amount is left.

(b) When is  $P = 0.50P_0$ ?  $0.50P_0 = P_0 e^{-0.2 \ln(0.90)t} \Rightarrow t = \frac{5 \ln(0.50)}{\ln(0.90)} \approx 32.9$  hours.

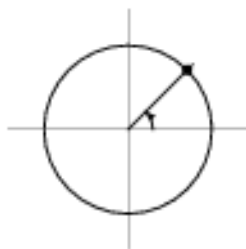
(c)



2. Section 1.5: 3, 4, 7, 9, 10, 13, 14, 17, 31, 30, 42.

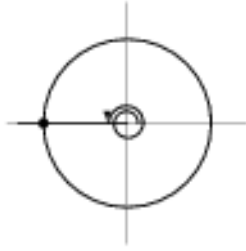
**Solution:**

3.



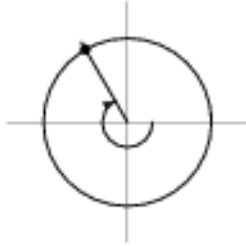
So  $\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$  or is positive,  $\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$  or is positive, and  $\tan(\frac{\pi}{4}) = 1$  or is positive.

4.



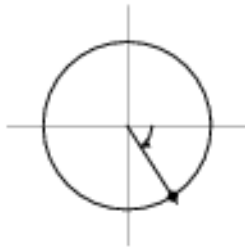
So  $\sin(3\pi) = 0$ ,  $\cos(3\pi) = -1$  or is negative, and  $\tan(3\pi) = 0$ .

7.



So  $\sin(\frac{-4\pi}{3}) = \frac{\sqrt{3}}{2}$  or is positive,  $\cos(\frac{-4\pi}{3}) = \frac{-1}{2}$  or is negative, and  $\tan(\frac{-4\pi}{3}) = -\sqrt{3}$  or is negative.

9.  $-1$  radian is  $(-180/\pi)^\circ \approx -57^\circ$ .

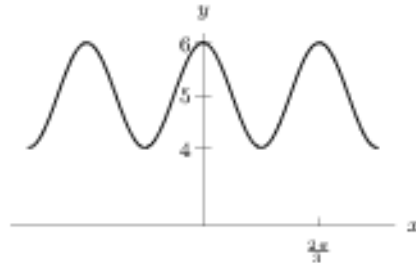


As in 4th quadrant,  $\cos(-1)$  is positive and  $\sin(-1)$  and  $\tan(-1)$  are negative.

10.  $\cos(-\frac{\pi}{5}) = \cos(\frac{\pi}{5}) = 0.809$ .

13. (a) Amplitude = 1, (b) Period =  $2\pi/3$ ,

(c)



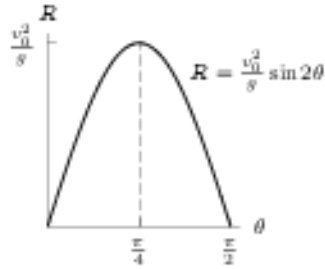
14. Period =  $2\pi/3$ ; amplitude = 7.

17. Period =  $2\pi/\pi = 2$ ; amplitude = 0.1.

31.  $\tan(5x) = 2$ ,  $5x \approx 1.11$ ,  $x \approx 0.2$ .

30.  $\cos(2x + 1) = 1/2$ ,  $2x + 1 = \pi/3$ ,  $x = \pi/6 - 1/2$ .

42.



The maximum range happens when  $\sin(2\theta)$  is made to be as big as possible, ie.  $\sin(2\theta) = 1$ . So have  $2\theta = \pi/2$  or  $\theta = \pi/4$ . And when  $\theta = \pi/4$ ,  $R = v_0^2/g$ .

3. Section 1.6: 1, 5 (I) (II) (V), 6, 8, 9, 10, 14.

**Solution:**

1. Exponential growth always dominates power growth as  $x \rightarrow \infty$ , so  $10 \cdot 2^x$  is larger.
5. (I) Minimum degree is 3 as 2 turning points; leading coefficient is negative because  $y \rightarrow -\infty$  as  $x \rightarrow \infty$ .  
(II) Minimum degree is 4 as 3 turning points; leading coefficient is positive because  $y \rightarrow \infty$  as  $x \rightarrow \infty$ .  
(V) Minimum degree is 5 as 4 turning points; leading coefficient is positive because  $y \rightarrow \infty$  as  $x \rightarrow \infty$ .
6. (a)  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$ ;  $f(x) \rightarrow -\infty$  as  $x \rightarrow \infty$ .  
(b)  $f(x) \rightarrow 3/2$  as  $x \rightarrow -\infty$ ;  $f(x) \rightarrow 3/2$  as  $x \rightarrow \infty$ .  
(c)  $f(x) \rightarrow 0$  as  $x \rightarrow -\infty$ ;  $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$ .
8. One possible formula is  $f(x) = -x(x+3)(x-4)$ .
9. One possible formula is  $f(x) = (x+2)(x-1)(x-3)(x-5)$ .
10. One possible formula is  $f(x) = -(x+2)(x-2)^2(x-5)$ .
14. (a) II, III (b) (I) (c) II, III (d) None (e) III.