

Solutions to Tutorials 13/03/08,14/03/08 and 17/03/08,18/03/08

MATH1111: Introduction to Calculus

Semester 1, 2008

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 1.4: 3, 4, 6, 9, 10, 11, 43 (a) (b) (c).

Solution:

3. $5A^2$.

4. $2AB$.

6. $2A \ln(e) + 3e \ln(B) = 2A + 3e \ln(B)$.

9. $\frac{20}{50} = (1.04)^x$, $\ln(2/5) = x \ln(1.04)$, $x = \frac{\ln(0.4)}{\ln(1.04)}$.

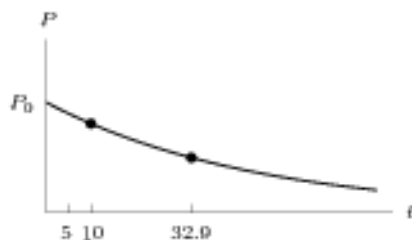
10. $\ln(4) + x \ln(3) = \ln(7) + x \ln(5)$, $x(\ln(3) - \ln(5)) = \ln(7) - \ln(4)$, $x = \frac{\ln(7/4)}{\ln(3/5)}$.

11. $\ln(2^x) = \ln(e^{x+1})$, $x \ln(2) = (x + 1) \ln(e)$, $x \ln(2) = x + 1$, $x(\ln(2) - 1) = 1$, $x = \frac{1}{\ln(2)-1}$.

43. (a) The decay follows the equation $P = P_0 e^{-5k}$, and 10% of the pollution is removed after 5 hours (meaning 90% is left). Therefore, $0.90P_0 = P_0 e^{-5k}$ or $k = -0.2 \ln(0.90)$. Thus, after 10 hours $P = P_0 e^{-10 \times (-0.2 \ln(0.90))}$ or $P = 0.81P_0$. So, 81% of the original amount is left.

(b) When is $P = 0.50P_0$? $0.50P_0 = P_0 e^{-0.2 \ln(0.90)t} \Rightarrow t = \frac{5 \ln(0.50)}{\ln(0.90)} \approx 32.9$ hours.

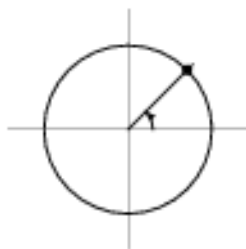
(c)



2. Section 1.5: 3, 4, 7, 9, 10, 13, 14, 17, 31, 30, 42.

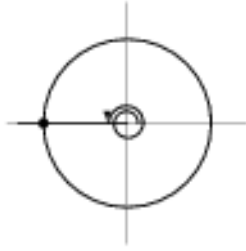
Solution:

3.



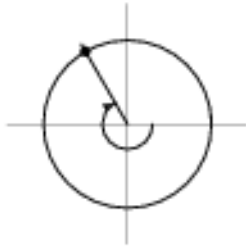
So $\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ or is positive, $\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ or is positive, and $\tan(\frac{\pi}{4}) = 1$ or is positive.

4.



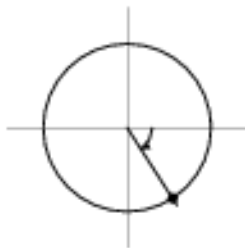
So $\sin(3\pi) = 0$, $\cos(3\pi) = -1$ or is negative, and $\tan(3\pi) = 0$.

7.



So $\sin(-\frac{4\pi}{3}) = \frac{\sqrt{3}}{2}$ or is positive, $\cos(-\frac{4\pi}{3}) = -\frac{1}{2}$ or is negative, and $\tan(-\frac{4\pi}{3}) = -\sqrt{3}$ or is negative.

9. -1 radian is $(-180/\pi)^\circ \approx -57^\circ$.

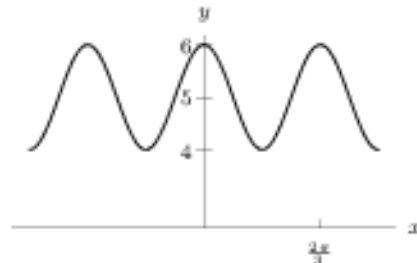


As in 4th quadrant, $\cos(-1)$ is positive and $\sin(-1)$ and $\tan(-1)$ are negative.

10. $\cos(-\frac{\pi}{5}) = \cos(\frac{\pi}{5}) = 0.809$.

13. (a) Amplitude = 1, (b) Period = $2\pi/3$,

(c)



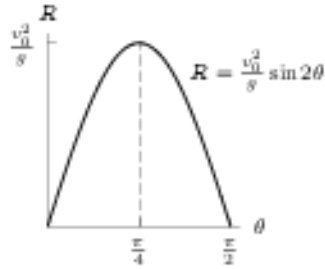
14. Period = $2\pi/3$; amplitude = 7.

17. Period = $2\pi/\pi = 2$; amplitude = 0.1.

31. $\tan(5x) = 2$, $5x \approx 1.11$, $x \approx 0.2$.

30. $\cos(2x + 1) = 1/2$, $2x + 1 = \pi/3$, $x = \pi/6 - 1/2$.

42.



The maximum range happens when $\sin(2\theta)$ is made to be as big as possible, ie. $\sin(2\theta) = 1$. So have $2\theta = \pi/2$ or $\theta = \pi/4$. And when $\theta = \pi/4$, $R = v_0^2/g$.

3. Section 1.6: 1, 5 (I) (II) (V), 6, 8, 9, 10, 14.

Solution:

1. Exponential growth always dominates power growth as $x \rightarrow \infty$, so $10 \cdot 2^x$ is larger.
5. (I) Minimum degree is 3 as 2 turning points; leading coefficient is negative because $y \rightarrow -\infty$ as $x \rightarrow \infty$.
(II) Minimum degree is 4 as 3 turning points; leading coefficient is positive because $y \rightarrow \infty$ as $x \rightarrow \infty$.
(V) Minimum degree is 5 as 4 turning points; leading coefficient is positive because $y \rightarrow \infty$ as $x \rightarrow \infty$.
6. (a) $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$; $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$.
(b) $f(x) \rightarrow 3/2$ as $x \rightarrow -\infty$; $f(x) \rightarrow 3/2$ as $x \rightarrow \infty$.
(c) $f(x) \rightarrow 0$ as $x \rightarrow -\infty$; $f(x) \rightarrow \infty$ as $x \rightarrow \infty$.
8. One possible formula is $f(x) = -x(x+3)(x-4)$.
9. One possible formula is $f(x) = (x+2)(x-1)(x-3)(x-5)$.
10. One possible formula is $f(x) = -(x+2)(x-2)^2(x-5)$.
14. (a) II, III (b) (I) (c) II, III (d) None (e) III.