

Solutions to Tutorials 20/03/08 and 31/03/08,1/04/08

MATH1111: Introduction to Calculus

Semester 1, 2008

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 2.1: 1, 3, 18, 21.

Solution:

1. Average velocity = $\frac{400-135}{5-2} = \frac{265}{3}$ km/hr.

3. (a) (i) Average velocity = $\frac{f(1.1)-f(1)}{1.1-1} = \frac{7.84-7}{0.1} = 8.4$ m/s.

(ii) Average velocity = $\frac{f(1.01)-f(1)}{1.01-1} = \frac{7.0804-7}{0.01} = 8.04$ m/s.

(iii) Average velocity = $\frac{f(1.001)-f(1)}{1.001-1} = \frac{7.008004-7}{0.001} = 8.004$ m/s.

(b) From part (a) we see that as our interval around $t = 1$ shrinks it appears as though our average velocity gets closer to 8. Hence we can estimate that the instantaneous velocity at $t = 1$ is 8 m/s.

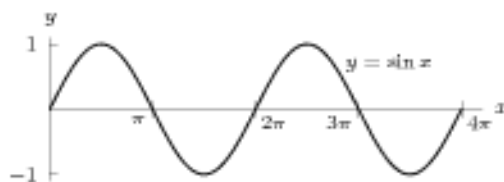
18. $\lim_{h \rightarrow 0} \frac{(2+h)^2-4}{h} = \lim_{h \rightarrow 0} \frac{4+4h+4h^2-4}{h} = \lim_{h \rightarrow 0} (4+h) = 4.$

21. $\lim_{h \rightarrow 0} \frac{(3+h)^2-(3-h)^2}{2h} = \lim_{h \rightarrow 0} \frac{9+6h+h^2-9+6h-h^2}{2h} = \lim_{h \rightarrow 0} \frac{12h}{2h} = 6.$

2. Section 2.2: 5, 8, 9, 10, 14.

Solution:

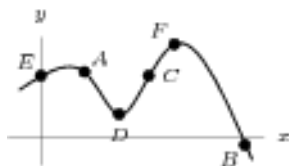
5.



Since $\sin(x)$ is decreasing for values near $x = 3\pi$, its derivative at $x = 3\pi$ is negative.

8. Since $f'(x) = 0$ where the graph is horizontal, $f'(x) = 0$ at $x = d$. The derivative is positive at points b and c , but the graph is steeper at $x = c$. Thus $f'(x) = 0.5$ at $x = b$ and $f'(x) = 2$ at $x = c$. Finally, the derivative is negative at points a and e but the graph is steeper at $x = e$. Thus, $f'(x) = -0.5$ at $x = a$ and $f'(x) = -2$ at $x = e$. Thus, we have $f'(d) = 0$, $f'(b) = 0.5$, $f'(c) = 2$, $f'(a) = -0.5$, $f'(e) = -2$.

9. One possible choice of points is shown below.



10. (a) The curve is concave down, so the average rate of change between $x = 1$ and $x = 3$ is greater than that between $x = 3$ and $x = 5$. (b) f is increasing, so $f(5)$ is greater. (c) f' is decreasing, so $f'(1)$ is greater.

14.

