

**Solutions to Tutorials 23/03/09,24/03/09 and 26/03/09,27/03/09**

MATH1111: Introduction to Calculus

Semester 1, 2009

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

Lecturer: Clio Cresswell

Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 2.1: 1, 3, 18, 21.

**Solution:**

1. Average velocity =  $\frac{400-135}{5-2} = \frac{265}{3}$  km/hr.

3. (a) (i) Average velocity =  $\frac{f(1.1)-f(1)}{1.1-1} = \frac{7.84-7}{0.1} = 8.4$  m/s.

(ii) Average velocity =  $\frac{f(1.01)-f(1)}{1.01-1} = \frac{7.0804-7}{0.01} = 8.04$  m/s.

(iii) Average velocity =  $\frac{f(1.001)-f(1)}{1.001-1} = \frac{7.008004-7}{0.001} = 8.004$  m/s.

(b) From part (a) we see that as our interval around  $t = 1$  shrinks it appears as though our average velocity gets closer to 8. Hence we can estimate that the instantaneous velocity at  $t = 1$  is 8 m/s.

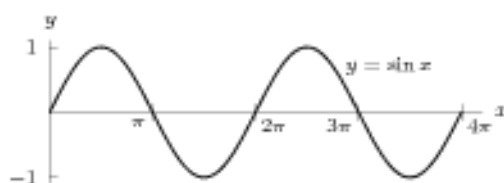
18.  $\lim_{h \rightarrow 0} \frac{(2+h)^2-4}{h} = \lim_{h \rightarrow 0} \frac{4+4h+4h^2-4}{h} = \lim_{h \rightarrow 0} (4+h) = 4.$

21.  $\lim_{h \rightarrow 0} \frac{(3+h)^2-(3-h)^2}{2h} = \lim_{h \rightarrow 0} \frac{9+6h+h^2-9+6h-h^2}{2h} = \lim_{h \rightarrow 0} \frac{12h}{2h} = 6.$

2. Section 2.2: 5, 8, 9, 10, 14.

**Solution:**

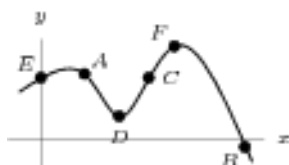
5.



Since  $\sin(x)$  is decreasing for values near  $x = 3\pi$ , its derivative at  $x = 3\pi$  is negative.

8. Since  $f'(x) = 0$  where the graph is horizontal,  $f'(x) = 0$  at  $x = d$ . The derivative is positive at points  $b$  and  $c$ , but the graph is steeper at  $x = c$ . Thus  $f'(x) = 0.5$  at  $x = b$  and  $f'(x) = 2$  at  $x = c$ . Finally, the derivative is negative at points  $a$  and  $e$  but the graph is steeper at  $x = e$ . Thus,  $f'(x) = -0.5$  at  $x = a$  and  $f'(x) = -2$  at  $x = e$ . Thus, we have  $f'(d) = 0$ ,  $f'(b) = 0.5$ ,  $f'(c) = 2$ ,  $f'(a) = -0.5$ ,  $f'(e) = -2$ .

9. One possible choice of points is shown below.



10. (a) The curve is concave down, so the average rate of change between  $x = 1$  and  $x = 3$  is greater than that between  $x = 3$  and  $x = 5$ . (b)  $f$  is increasing, so  $f(5)$  is greater. (c)  $f'$  is decreasing, so  $f'(1)$  is greater.

14.

