

Solutions to Tutorials 07/04/08,08/04/08 and 10/04/08,11/04/08

MATH1111: Introduction to Calculus

Semester 1, 2008

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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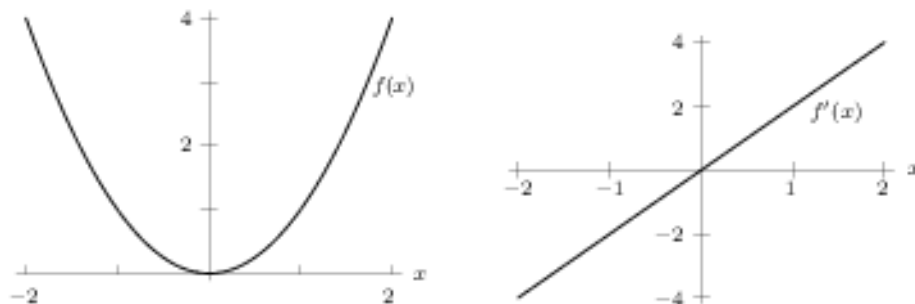
Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 2.3: 15, 17, 23, 25, 27, 29, 35, 36, 37, 42.

**Solution:**

15.  $g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 3 - (2x^2 - 3)}{h} = \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h} = \lim_{h \rightarrow 0} (4x + 2h) = 4x.$

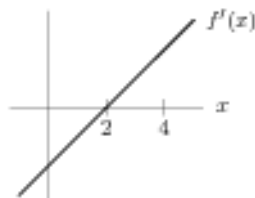
17.



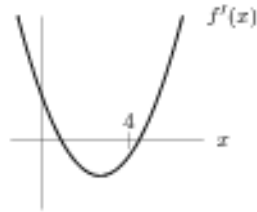
23. This is a line with slope 1, so the derivative is the constant function  $f'(x) = 1$ . The graph is the horizontal line  $y = 1$ .



25.



27.

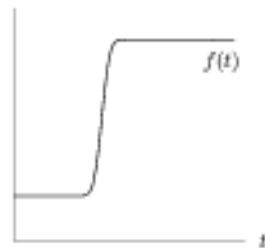


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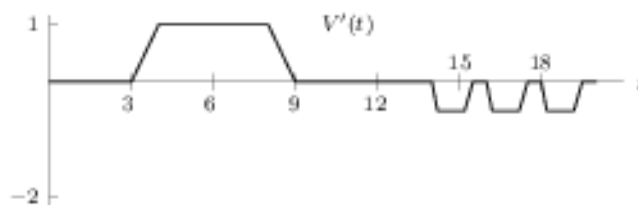


35. (a) II, (b) I, (c) III.

36. On intervals where  $f' = 0$ ,  $f$  is constant. On the small interval where  $f' > 0$ ,  $f$  is increasing. Where  $f'$  hits the top of its spike,  $f$  is increasing quite sharply. So  $f$  should be constant for a while, have a sudden increase, and then be constant again. A possible graph for  $f$  is as follows:



37. (a)  $t = 3$ , (b)  $t = 9$ , (c)  $t = 14$ , (d) Instead of one cup shape for  $14 \leq t \leq 17$ , there would be a number of smaller cup shapes for  $t \geq 14$ , possibly:



42. (a) Where  $f'$  is positive, so for  $x_1 < x < x_3$ . (b) Where  $f'$  is negative, so for  $0 < x < x_1$  and  $x_3 < x < x_5$ .

2. Section 2.4: 1, 3, 14.

**Solution:**

1. (a) As the cup of coffee cools, the temperature decreases, so  $f'(t)$  is negative. (b)  $f'(t) = \frac{dH}{dt}$ , so the units are degrees Celsius per minute.  $f'(20)$  represents the rate at which the coffee is cooling (in degrees per minute) 20 mins after the cup is put on the counter.

3. (a)  $f(200) = 350$  means it costs \$350 to produce 200 gallons of ice cream. (b)  $f'(200) = 1.4$  means that when the number of gallons produced is 200, costs are increasing by about \$1.40 per gallon.

14. (a)  $f(140) = 120$  means a patient weighing 140 pounds should receive a dose of 120 mg.  $f'(140) = 3$  means if the patient increases their weight by 1 pound (from 140), the dose should be increased by 3 mg. (b)  $f(140) = 120$  and from here the dose goes up by 3 mg for each pound. So a 145 pound patient should approximately get an additional  $3 \times 5 = 15$  mg. Hence the approximate dose would be  $120 + 15 = 135$  mg.

3. Section 2.5: 1, 7, 8, 9, 10, 20.

**Solution:**

1. (a) At  $x = 2$ , the graph is below the  $x$ -axis so  $f(2) < 0$ . (b) At  $x = 2$ ,  $f(x)$  is decreasing so  $f'(2) < 0$ . (c) At  $x = 2$ ,  $f(x)$  is concave up so  $f''(x) > 0$ .

7.  $f'(x) > 0$ ,  $f''(x) > 0$ .

8.  $f'(x) = 0$ ,  $f''(x) = 0$ .

9.  $f'(x) < 0$ ,  $f''(x) = 0$ .

10.  $f'(x) < 0$ ,  $f''(x) > 0$ .

20. (a) At  $t_3$ ,  $t_4$ , and  $t_5$ , as the graph is above the  $t$ -axis. (b) At  $t_2$  and  $t_3$  as the graph has an upward slope. (c) At  $t_1$ ,  $t_2$ , and  $t_5$ , as the graph is concave up. (d) At  $t_1$ ,  $t_4$ , and  $t_5$ , as the graph has a downward slope. (e) At  $t_3$  and  $t_4$  as the graph is concave down.

4. Section 3.1: 1, 4, 6, 10, 12, 15, 18, 19, 21, 24, 28, 30, 34, 37, 52, 53, 57.

**Solution:**

1.  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{7-7}{h} = 0$ .

4.  $y' = 12x^{11}$ .

6.  $y' = 11x^{-12}$ .

10.  $y' = -\frac{3}{4}x^{-7/4}$ .

12.  $g'(x) = -5t^{-6}$ .

15.  $y = x^{1/2}$ ,  $y' = \frac{1}{2}x^{-1/2}$ .

18.  $f(x) = x^{-3/2}$ ,  $f'(x) = -\frac{3}{2}x^{-5/2}$ .

19.  $f'(x) = ex^{e-1}$ .

21.  $f'(t) = 6t - 4$ .

24.  $f'(x) = 20x^3 - \frac{2}{x^3}$ .

28.  $y' = 6t - \frac{6}{t^{3/2}} + \frac{2}{t^3}$ .

30.  $y = 2t^{3/2} + t^2$ ,  $y' = 3t^{1/2} + 2t$ .

34.  $f(z) = \frac{z}{3} + \frac{1}{3}z^{-1}$ ,  $f'(z) = \frac{1}{3} - \frac{1}{3}z^{-2}$ .

37.  $j'(x) = \frac{3x^2}{a} + \frac{2ax}{b} - c$ .

52.  $f'(x) = -8 + 2\sqrt{2}x$ ,  $f'(r) = -8 + 2\sqrt{2}r = 4$ , so  $r = \frac{12}{2\sqrt{2}} = 3\sqrt{2}$ .

53.  $f'(x) = 6x^2 - 4x$  so  $f'(1) = 2$ . Hence the equation of the tangent line at  $(1, 1)$  is  $(y - 1) = 2(x - 1)$  or  $y = 2x - 1$ .

57.  $f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$ .  $f$  decreasing when  $f' < 0$ , i.e. when  $x < 3$  (with  $x \neq 0$ ). For concavity need to consider  $f''$ .  $f''(x) = 12x^2 - 24x = 12x(x - 2)$ . Concave up means  $f'' > 0$  and this occurs when  $x < 0$  or  $x > 2$ . Thus the intervals on which  $f$  is both decreasing and concave up are:  $x < 0$  and  $2 < x < 3$ .