

Solutions to Tutorials 30/03/09,31/03/09 and 02/04/09,03/04/09

MATH1111: Introduction to Calculus

Semester 1, 2009

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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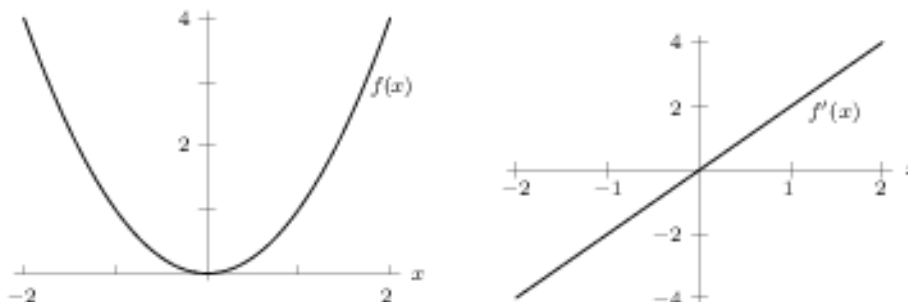
Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 2.3: 15, 17, 23, 25, 27, 29, 35, 36, 37, 42.

Solution:

15. $g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 3 - (2x^2 - 3)}{h} = \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h} = \lim_{h \rightarrow 0} (4x + 2h) = 4x.$

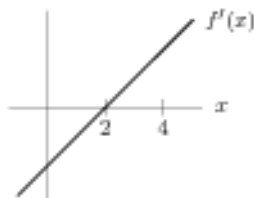
17.



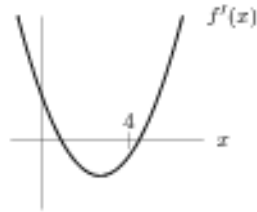
23. This is a line with slope 1, so the derivative is the constant function $f'(x) = 1$. The graph is the horizontal line $y = 1$.



25.



27.

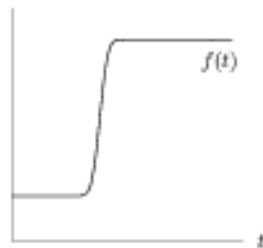


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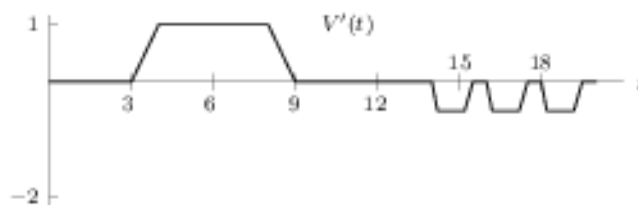


35. (a) II, (b) I, (c) III.

36. On intervals where $f' = 0$, f is constant. On the small interval where $f' > 0$, f is increasing. Where f' hits the top of its spike, f is increasing quite sharply. So f should be constant for a while, have a sudden increase, and then be constant again. A possible graph for f is as follows:



37. (a) $t = 3$, (b) $t = 9$, (c) $t = 14$, (d) Instead of one cup shape for $14 \leq t \leq 17$, there would be a number of smaller cup shapes for $t \geq 14$, possibly:



42. (a) Where f' is positive, so for $x_1 < x < x_3$. (b) Where f' is negative, so for $0 < x < x_1$ and $x_3 < x < x_5$.

2. Section 2.4: 1, 3, 14.

Solution:

1. (a) As the cup of coffee cools, the temperature decreases, so $f'(t)$ is negative. (b) $f'(t) = \frac{dH}{dt}$, so the units are degrees Celsius per minute. $f'(20)$ represents the rate at which the coffee is cooling (in degrees per minute) 20 mins after the cup is put on the counter.

3. (a) $f(200) = 350$ means it costs \$350 to produce 200 gallons of ice cream. (b) $f'(200) = 1.4$ means that when the number of gallons produced is 200, costs are increasing by about \$1.40 per gallon.

14. (a) $f(140) = 120$ means a patient weighing 140 pounds should receive a dose of 120 mg. $f'(140) = 3$ means if the patient increases their weight by 1 pound (from 140), the dose should be increased by 3 mg. (b) $f(140) = 120$ and from here the dose goes up by 3 mg for each pound. So a 145 pound patient should approximately get an additional $3 \times 5 = 15$ mg. Hence the approximate dose would be $120 + 15 = 135$ mg.

3. Section 2.5: 1, 7, 8, 9, 10, 20.

Solution:

1. (a) At $x = 2$, the graph is below the x -axis so $f(2) < 0$. (b) At $x = 2$, $f(x)$ is decreasing so $f'(2) < 0$. (c) At $x = 2$, $f(x)$ is concave up so $f''(x) > 0$.

7. $f'(x) > 0, f''(x) > 0$.

8. $f'(x) = 0, f''(x) = 0$.

9. $f'(x) < 0, f''(x) = 0$.

10. $f'(x) < 0, f''(x) > 0$.

20. (a) At t_3, t_4 , and t_5 , as the graph is above the t -axis. (b) At t_2 and t_3 as the graph has an upward slope. (c) At t_1, t_2 , and t_5 , as the graph is concave up. (d) At t_1, t_4 , and t_5 , as the graph has a downward slope. (e) At t_3 and t_4 as the graph is concave down.

4. Section 3.1: 1, 4, 6, 10, 12, 15, 18, 19, 21, 24, 28, 30, 34, 37, 52, 53, 57.

Solution:

1. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{7-7}{h} = 0$.

4. $y' = 12x^{11}$.

6. $y' = 11x^{-12}$.

10. $y' = -\frac{3}{4}x^{-7/4}$.

12. $g'(x) = -5t^{-6}$.

15. $y = x^{1/2}, y' = \frac{1}{2}x^{-1/2}$.

18. $f(x) = x^{-3/2}, f'(x) = -\frac{3}{2}x^{-5/2}$.

19. $f'(x) = ex^{e-1}$.

21. $f'(t) = 6t - 4$.

24. $f'(x) = 20x^3 - \frac{2}{x^3}$.

28. $y' = 6t - \frac{6}{t^{3/2}} + \frac{2}{t^3}$.

30. $y = 2t^{3/2} + t^2, y' = 3t^{1/2} + 2t$.

34. $f(z) = \frac{z}{3} + \frac{1}{3}z^{-1}, f'(z) = \frac{1}{3} - \frac{1}{3}z^{-2}$.

37. $j'(x) = \frac{3x^2}{a} + \frac{2ax}{b} - c$.

52. $f'(x) = -8 + 2\sqrt{2}x, f'(r) = -8 + 2\sqrt{2}r = 4$, so $r = \frac{12}{2\sqrt{2}} = 3\sqrt{2}$.

53. $f'(x) = 6x^2 - 4x$ so $f'(1) = 2$. Hence the equation of the tangent line at $(1, 1)$ is $(y - 1) = 2(x - 1)$ or $y = 2x - 1$.

57. $f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$. f decreasing when $f' < 0$, i.e. when $x < 3$ (with $x \neq 0$). For concavity need to consider f'' . $f''(x) = 12x^2 - 24x = 12x(x - 2)$. Concave up means $f'' > 0$ and this occurs when $x < 0$ or $x > 2$. Thus the intervals on which f is both decreasing and concave up are: $x < 0$ and $2 < x < 3$.