

Solutions to Tutorials 14/04/08,15/04/08 and 17/04/08,18/04/08

MATH1111: Introduction to Calculus

Semester 1, 2008

Web Page: <http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/>

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Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 3.2: 2, 11, 16.

Solution:

2. $y' = 10t + 4e^t$.

11. $z' = (\ln(4))e^x$.

16. $f'(x) = ex^{e-1}$.

2. Section 3.3: 1, 3, 6, 10, 12, 17, 18, 22, 40, 44, 53.

Solution:

1. By the product rule: $f'(x) = 2x(x^3 + 5) + x^2(3x^2) = 5x^4 + 10x$. By multiplying out first: $f(x) = x^5 + 5x^2$, $f'(x) = 5x^4 + 10x$. Get the same result and should!

3. $f'(x) = xe^x + e^x$.

6. $y' = 2te^t + (t^2 + 3)e^t$.

10. $y' = (3t^2 - 14t)e^t + (t^3 - 7t^2 + 1)e^t$.

12. $g'(x) = \frac{50xe^x - 25x^2e^x}{e^{2x}} = \frac{50x - 25x^2}{e^x}$.

17. $z' = \frac{3(5t+2) - (3t+1)5}{(5t+2)^2} = \frac{1}{(5t+2)^2}$.

18. $z' = \frac{(2t+5)(t+3) - (t^2+5t+2)}{(t+3)^2} = \frac{t^2+6t+13}{(t+3)^2}$.

22. $y' = \frac{\frac{1}{2}t^{-1/2}(t^2+1) - \sqrt{t}(2t)}{(t^2+1)^2}$.

40. $f'(x) = 3(2x - 5) + 2(3x + 8) = 12x + 1$, $f''(x) = 12$.

44. $f'(x) = e^x + xe^x$ and $f''(x) = e^x + e^x + xe^x = (2+x)e^x$. $f(x)$ is concave up when $f''(x) > 0$ and that happens when $x > -2$.

53. (a) $f(140) = 15,000$ means 15,000 skateboards are sold when the cost is \$140 per board. $f'(140) = -100$ means once skateboards hit the price of \$140 each, then total sales will decrease by 100 boards if the price increases by a dollar.

(b) $\frac{dR}{dp} = \frac{d}{dp}(p \cdot q) = \frac{d}{dp}(p \cdot f(p)) = f(p) + pf'(p)$. At $p = 140$, $\frac{dR}{dp} = f(140) + 140f'(140) = 15000 + 140(-100) = 1000$.

(c) From (b) we see that at $p = 140$ $\frac{dR}{dp} > 0$. Hence the revenue will increase by about \$1000 if the price is raised by \$1 to \$141.

3. Section 3.4: 1, 2, 6, 8, 9, 11, 12, 17, 18, 33, 36, 63, 69, 66.

Solution:

1. $f'(x) = 99(x + 1)^{98}$.

2. $w' = 200t(t^2 + 1)^{99}$.

6. $\frac{d}{dx}(\sqrt{e^x + 1}) = \frac{1}{2}(e^x + 1)^{-1/2} \frac{d}{dx}(e^x + 1) = \frac{e^x}{2\sqrt{e^x + 1}}$.

8. $h'(w) = 5(w^4 - 2w)^4(4w^3 - 2)$.
9. $w'(r) = \frac{1}{2}(r^4 + 1)^{-1/2}4r^3 = \frac{2r^3}{\sqrt{r^4+1}}$.
11. $f'(x) = 2e^{2x}(x^2 + 5^x) + e^{2x}(2x + (\ln(5))5^x) = e^{2x}[2x^2 + 2x + (\ln(5) + 2)5^x]$.
12. $f'(t) = 3e^{3t}$.
17. $f'(t) = e^{5-2t} + te^{5-2t}(-2) = e^{5-2t}(1 - 2t)$.
18. $p'(t) = 4e^{4t+2}$.
33. $y' = 2\left(\frac{x^2+2}{3}\right)\left(\frac{2x}{3}\right) = \frac{4}{9}x(x^2 + 2)$.
36. $y' = \frac{-(3e^{3x}+2x)}{(e^{3x}+x^2)^2}$.
63. $y' = -2xe^{-x^2}$ and $y'' = -2e^{-x^2} - 2xe^{-x^2}(-2x) = \frac{4x^2-2}{e^{2x}}$. The graph is concave down when $f''(x) < 0$ and that happens when $4x^2 - 2 < 0$ or $x^2 < \frac{1}{2}$, i.e. when $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$.
69. With $x = (2t + 5)^{1/3}$, $3x^2 \frac{dx}{dt} = 3(2t + 5)^{2/3} \frac{1}{3}(2t + 5)^{-2/3} 2 = 2$. So $x = (2t + 5)^{1/3}$ is a solution to the equation.
66. (a) The rate of growth of the population is $P'(t)$. $P'(t)$ is proportional to $P(t)$ so $P'(t) = kP(t)$. (b) $P(t) = Ae^{kt}$ means $P'(t) = kAe^{kt} = kP(t)$.