The University of Sydney
School of Mathematics and Statistics

## Solutions to Tutorials 20/04/09,21/04/09 and 23/04/09,24/04/09

MATH1111: Introduction to Calculus — Semester 1, 2009

Web Page: `http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/`
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Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

**1.** Section 3.6: 2, 3, 7, 28.

***Solution:***

2. $f'(x) = \dfrac{-1}{1-x}$.

3. $f(x) = 2x \ln e = 2x$ so $f'(x) = 2$.

7. $f'(x) = \dfrac{e^x}{e^x + 1}$.

28. $f'(x) = \dfrac{(1+\ln x) - x(\frac{1}{x})}{(1+\ln x)^2} = \dfrac{\ln x}{(1+\ln x)^2}$.

**2.** Section 3.2: 4, 20, 22, 36.

***Solution:***

4. $f'(x) = 12e^x + (\ln(11))11^x$.

20. $f'(x) = (\ln(\pi))\pi^x + \pi x^{\pi-1}$.

22. Using Chain Rule $f'(x) = e^{1+x}$.

36. With $P_0 = 1$ have $P = (1.05)^t$, and then $P' = (\ln(1.05))(1.05)^t$. When $t = 10$, $P' = (\ln(1.05))(1.05)^{10} \approx 0.0795$. That's an increase of 7.95 cents per year.

**3.** Section 3.3: 5, 9.

***Solution:***

5. $y' = \frac{1}{2}x^{-1/2}2^x + \sqrt{x}(\ln(2))2^x$.

9. $f'(y) = (\ln(4))4^y(2-y^2) + 4^y(-2y)$.

**4.** Section 3.4: 11.

***Solution:***

11. $f'(x) = 2e^{2x}(x^2+5^x) + e^{2x}(2x + (\ln(5))5^x) = e^{2x}[2x^2 + 2x + (\ln(5)+2)5^x]$.

**5.** Section 3.5: 2, 3, 4, 6, 8, 9, 10, 13, 18, 19, 24, 26, 33, 41, 46.

***Solution:***

2. $r'(\theta) = \cos(\theta) - \sin(\theta)$.

3. $s'(\theta) = -\sin(\theta)\sin(\theta) + \cos(\theta)\cos(\theta) = \cos^2(\theta) - \sin^2(\theta)$.

4. $z' = -4\sin(4\theta)$.

6. $g'(x) = \cos(2-3x)\frac{d}{dx}(2-3x) = -3\cos(2-3x)$.

8. $g'(\theta) = 2\sin(2\theta)\cos(2\theta)2 - \pi = 4\sin(2\theta)\cos(2\theta) - \pi$.

9. $f'(x) = 2x\cos(x) - x^2\sin(x)$.

10. $w' = e^t\cos(e^t)$.

13. $z' = e^{\cos(\theta)} - \theta(\sin(\theta))e^{\cos(\theta)}$.

18. $f'(x) = [-\sin(\sin(x))](\cos(x))$.

19. $f'(x) = \cos(x)\sec^2(\sin(x))$.

24. $z' = \dfrac{\cos(t)}{2\sqrt{\sin(t)}}$.

26. $g'(z) = e^z\sec^2(e^z)$.

33. $y' = -2\cos(w)\sin(w) - \sin(w^2)2w = -2(\cos(w)\sin(w) + w\sin(w^2))$.

41. $\dfrac{dy}{dx} = -\sin(x)$, $\dfrac{d^2y}{dx^2} = -\cos(x)$, $\dfrac{d^3y}{dx^3} = \sin(x)$, $\dfrac{d^4y}{dx^4} = \cos(x)$, and so on. So derivatives of $y$ that are multiples of 4 will be $\cos(x)$. So $\dfrac{d^{48}y}{dx^{48}} = \cos(x)$, $\dfrac{d^{49}y}{dx^{49}} = -\sin(x)$, and $\dfrac{d^{50}y}{dx^{50}} = -\cos(x)$.

46. (a) $\dfrac{dV}{dt} = -120\pi \times 156\sin(120\pi t) = -18720\pi\sin(120\pi t)$ volts per second.
(b) $\dfrac{dV}{dt} = 0 \Rightarrow \sin(120\pi t) = 0$. This occurs whenever $120\pi t$ equals any multiple of $\pi$. There are therefore an infinite number of times $dV/dt$ can be zero.
(c) $\dfrac{dV}{dt} = -18720\pi\sin(120\pi t)$ and $-1 \le \sin(120\pi t) \le 1$. So the maximum rate of change occurs when $\sin(120\pi t) = -1$ and is $18720\pi$ volts per second.

**6.** Section 4.1: 4, 5, 6, 7, 21, 24, 27, 36, 38.

***Solution:***

4. $f'(x) = 10.2x^2(-0.4e^{-0.4x}) + 20.4xe^{-0.4x} = -4.08x^2e^{-0.4x} + 20.4xe^{-0.4x}$. When $f'(x) = 0$, $-4.08x^2e^{-0.4x} + 20.4xe^{-0.4x} = 0$ or $x(-4.08x + 20.4)e^{-0.4x} = 0 \Rightarrow x = 0, 5$. There are two critical points $(0, 0)$ and $(5, 255e^{-2})$.

5. $f'(x) = 4x^3 + 3x^2 - 6x$ and $f''(x) = 12x^2 + 6x - 6$. Solving $f''(x) = 0$ gives $x = -1$ and $x = 1/2$. These are two possible inflexion points. Before we test for the sign of $f''(x)$, locate possible critical points to make sure we test correctly. $f'(x) = 0 \Rightarrow x(4x^2 + 3x - 6) = 0 \Rightarrow x \approx -1.66, 0, 0.91$. Test: $f''(-1.2) > 0$ and $f''(-0.9) < 0$, so $x = -1$ is an inflexion point. Test: $f''(0.4) < 0$ and $f''(0.6) > 0$, so $x = 1/2$ is an inflexion point.

6. $g'(x) = (1 - x)e^{-x}$, so $x = 1$ is the only critical point. $g''(x) = (x - 2)e^{-x}$ and $g''(1) = -e^{-1} < 0$ so $(1, e^{-1})$ is a local maximum.

7. $h'(x) = 1 - \dfrac{1}{x^2}$, so the critical points occur at $x = \pm 1$. $h''(x) = \dfrac{2}{x^3}$ with $h''(1) > 0$ and $h''(-1) < 0$. So $(1, 2)$ is a local minimum and $(-1, -2)$ is a local maximum.

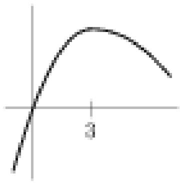
21. $f'(x) = 3x^2(1 - x)^4 - 4x^3(1 - x)^3 = x^2(1 - x)^3(3 - 7x)$, so the critical points occur at $x = 0, 1, \dfrac{3}{7}$. For $x < 0$: $x^2 > 0$, $1 - x > 0$, and $3 - 7x > 0$, so $f'(x) > 0$. For $0 < x < \dfrac{3}{7}$: $x^2 > 0$, $1 - x > 0$, and $3 - 7x > 0$, so $f'(x) > 0$. For $\dfrac{3}{7} < x < 1$: $x^2 > 0$, $1 - x > 0$, and $3 - 7x < 0$, so $f'(x) < 0$. For $x > 1$: $x^2 > 0$, $1 - x < 0$, and $3 - 7x < 0$, so $f'(x) > 0$. So $(0, 0)$ is neither a local maximum nor a local minimum, $(\dfrac{3}{7}, \dfrac{3^3 4^4}{7^7})$ is a local maximum, and $(1, 0)$ is a local minimum.

24. (a) Critical points occur when $f'(x) = 0$. Since $f'(x)$ changes sign between $x = 2$ and $x = 3$, between $x = 6$ and $x = 7$, and between $x = 9$ and $x = 10$, we expect critical points in those intervals. (b) $f'(2) > 0$ and $f'(3) < 0$ so we expect a local maximum for $2 < x < 3$. $f'(6) < 0$ and $f'(7) > 0$ so we expect a local minimum for $6 < x < 7$. $f'(9) > 0$ and $f'(10) < 0$ so we expect a local maximum for $9 < x < 10$.

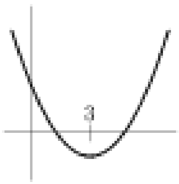
27. A local minimum at $(6, -5)$ implies that $f'(6) = 0$. $f'(x) = 2x + a$, $f'(6) = 12 + a = 0 \Rightarrow a = -12$. So now we know that $f(x) = x^2 - 12x + b$. Also $f(6) = -5$, so $6^2 - 12 \times 6 + b = -5 \Rightarrow b = 31$. So $f(x) = x^2 - 12x + 31$.

36. Since $f$ is differentiable everywhere, $f'$ must be zero (not undefined) at any critical points. So $f'(3) = 0$. Since $f$ has just one critical point, $f'$ may change sign only at $x = 3$. Thus $f$ is always increasing or decreasing for $x < 3$ and for $x > 3$.

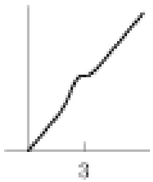
(a) $x = 3$ is a local maximum as $f$ is increasing when $x < 3$ and decreasing when $x > 3$.



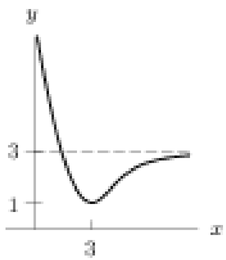

(b) $x = 3$ is a local minimum as $f$ heads to infinity to either side of $x = 3$.




(c) $x = 3$ is neither a local minimum or a local maximum as $f(1) < f(2) < f(4) < f(5)$.




(d) $x = 3$ is a local minimum as $f$ is decreasing to the left of $x = 3$ and must increase to the right as $f(3) = 1$ and eventually $f$ becomes close to 3.




38. Neither $B$ or $C$ is 0 where $A$ has its maxima and minimum. Therefore neither $B$ or $C$ is the derivative of $A$, so $A = f''$. $B$ could be the derivative of $C$ as where $C$ has a maximum $B$ is 0. However $C$ could not be the derivative of $B$ as $B$ is decreasing for some $x-$values and $C$ is never negative. Thus, $C = f$, $B = f'$, and $A = f''$.