THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Solutions to Tutorials 04/05/09,05/05/09 and 07/05/09,08/05/09

MATH1111: Introduction to Calculus — Semester 1, 2009

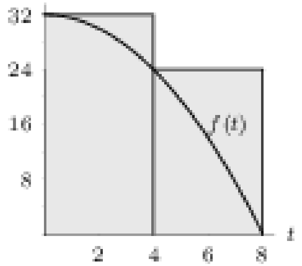
Web Page: `http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/`
Lecturer: Clio Cresswell

Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

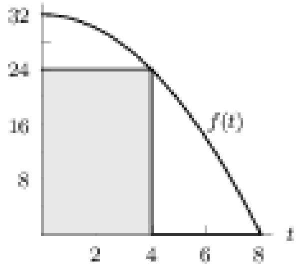
1. Section 5.2: 1, 19, 27.

Solution:

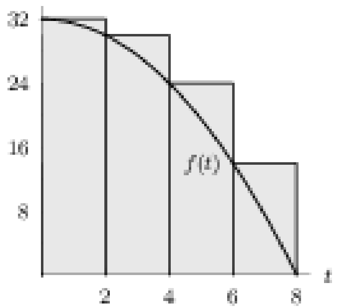
1. (a) Left-hand sum $= 32 \times 4 + 24 \times 4 = 224$.



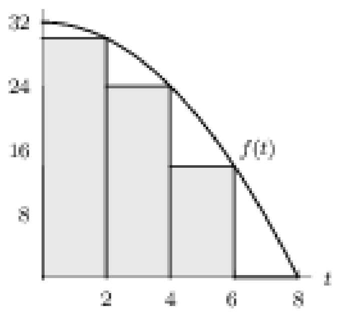
(b) Right-hand sum $= 24 \times 4 + 0 \times 4 = 96$.



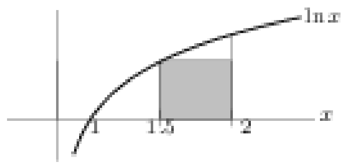
(c) Left-hand sum $= 32 \times 2 + 30 \times 2 + 24 \times 2 + 14 \times 2 = 200$.



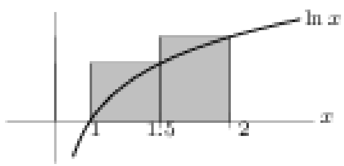
(d) Right-hand sum $= 30 \times 2 + 24 \times 2 + 14 \times 2 + 0 \times 2 = 136$.



19. (a) Left-hand sum $= f(1)\Delta x + f(1.5)\Delta x$. $\Delta x = \frac{2-1}{2} = 0.5$ so left-hand sum $= \ln(1)0.5 + \ln(1.5)0.5 = 0.5\ln(1.5)$.



(b) Right-hand sum $= f(1.5)\Delta x + f(2)\Delta x = \ln(1.5)0.5 + \ln(2)0.5$.



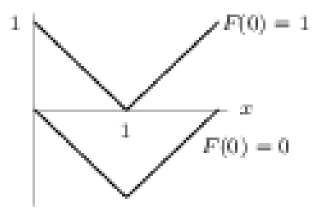
(c) The right-hand sum is an overestimate and the left-hand sum is an underestimate.

27. (a) $\int_a^b f(x)dx = 13$. (b) $\int_b^c f(x)dx = -2$. (c) $\int_a^c f(x)dx = 13 - 2 = 11$. (d) The graph of $|f(x)|$ is the same as the graph of $f(x)$ except that the part below the x-axis is reflected to be above it. Hence $\int_a^b |f(x)|dx = 13 + 2 = 15$.

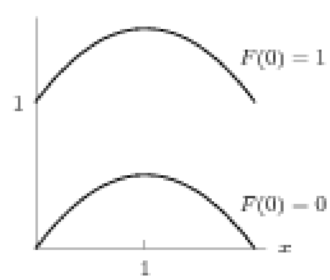
2. Section 6.1: 1, 2, 8, 10, 11.

Solution:

1.

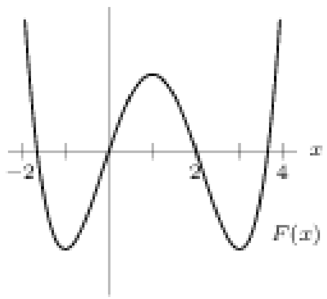


2.



8. By the Fundamental Theorem of Calculus, $P(1) - P(0) = \int_0^1 P'(t)dt$ or $P(1) = P(0) + \int_0^1 P'(t)dt = 2 - 1 \times 1 = 1$. Similarly, $P(2) - P(1) = \int_1^2 P'(t)dt$ or $P(2) = P(1) + \int_1^2 P'(t)dt = 1 - 1 \times 1 = 0$. And, $P(3) - P(2) = \int_2^3 P'(t)dt$ or $P(3) = P(2) + \int_2^3 P'(t)dt = 0 - \frac{1}{2} \times 1 \times 1 = -\frac{1}{2}$. And, $P(4) - P(3) = \int_3^4 P'(t)dt$ or $P(4) = P(3) + \int_3^4 P'(t)dt = -\frac{1}{2} + \frac{1}{2} \times 1 \times 1 = 0$. And, $P(5) - P(4) = \int_4^5 P'(t)dt$ or $P(5) = P(4) + \int_4^5 P'(t)dt = 0 + 1 \times 1 = 1$.

10. (a) Where $F'(x)$ or $f(x)$ equals 0, i.e. $x = -1, 1, 3$.
(b) $F(x)$ has a local minimum at $x = -1$, a local maximum at $x = 1$, and a local minimum at $x = 3$.
(c)



11. Note that since $f(x_1) = 0$ and $f'(x_1) < 0$, $(x_1, F(x_1))$ is a local maximum. Also since $f(x_3) = 0$ and $f'(x_3) > 0$, $(x_3, F(x_3))$ is a local minimum. And since $f'(x_2) = 0$ and f changes from decreasing to increasing about $x = x_2$, F has an inflection point at $x = x_2$.