THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Solutions to Tutorials 11/05/09,12/05/09 and 14/05/09,15/05/09

MATH1111: Introduction to Calculus — Semester 1, 2009

Web Page: http://www.maths.usyd.edu.au/u/UG/JM/MATH1111/
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Textbook questions refer to *Calculus: Single and Multivariable*, by Deborah Hughes-Hallett, Andrew M. Gleason, William G. McCallum *et al.*, John Wiley & Sons, 4th ed.

1. Section 6.2: 22, 24, 26, 28, 33, 40, 45, 49, 52, 54, 61, 62, 63, 67, 72, 74, 77, 86.

Solution:

22. $\int 6t dt = 3t^2 + C$.

24. $\int x^2 - 4x + 7dx = \frac{x^3}{3} - 2x^2 + 7x + C$.

26. $\int z + e^z dz = \frac{z^2}{2} + e^z + C$.

28. $\int \sin x + \cos x dx = -\cos x + \sin x + C$.

33. $F(x) = 3x + C$. $F(0) = 0 \Rightarrow 3 \times 0 + C = 0$ or $C = 0$. So $F(x) = 3x$.

40. $F(x) = -\cos x + C$. $F(0) = 0 \Rightarrow -\cos 0 + C = 0$ or $C = 1$. So $F(x) = -\cos x + 1$.

45. $\int t^2 + \frac{1}{t^2} dt = \frac{t^3}{3} - \frac{1}{t} + C$.

49. $\int 4t + 7dt = 2t^2 + 7t + C$.

52. $\int x + \frac{1}{\sqrt{x}} dx = \frac{x^2}{2} + 2x^{1/2} + C$.

54. $\int \pi + x^{11} dx = \pi x + \frac{x^{12}}{12} + C$.

61. $\int_1^3 \frac{1}{t} dt = [\ln|t|]_1^3 = \ln|3| - \ln|1| = \ln 3$.

62. $\int_0^{\pi/4} \sin x dx = [-\cos x]_0^{\pi/4} = -\cos\frac{\pi}{4} - (-\cos 0) = -\frac{\sqrt{2}}{2} + 1$.

63. $\int_0^2 3e^x dx = [3e^x]_0^2 = 3e^2 - 3e^0 = 3e^2 - 3$.

67. $\int_0^2 \frac{x^3}{3} + 2x dx = \left[\frac{x^4}{12} + x^2\right]_0^2 = \frac{4}{3} + 4 = \frac{16}{3}$.

72. Area$= \int_1^4 x^2 dx = \left[\frac{x^3}{3}\right]_1^4 = \frac{4^3}{3} - \frac{1^3}{3} = 21$.

74. $y = 0 \Rightarrow x = 0, 1$. So Area$= \int_0^1 x^2(1-x)^2 dx = \int_0^1 x^2(1-2x+x^2)dx = \int_0^1 x^2 - 2x^3 + x^4 dx = \left[\frac{x^3}{3} - \frac{2}{4}x^4 + \frac{x^5}{5}\right]_0^1 = \frac{1}{30}$.

77. The graph of e^x is always above the graph of $\cos x$ for $0 \le x \le 1$. Area$= \int_0^1 e^x - \cos x dx = [e^x - \sin x]_0^1 = e^1 - \sin 1 - e^0 + \sin 0 = e - 1 - \sin 1$.

86. Want to find $C(x)$. $C(x) = \int 4000 + 10x dx = 4000x + 5x^2 + K$ where K is a constant. The fixed costs are the ones that don't depend on depth of drilling i.e. x. So $C(0) = 1,000,000 = K$. So $C(x) = 4000x + 5x^2 + 1000000$.

2. Section 7.1: 3, 5, 6, 7, 10, 12, 16, 18, 20, 21, 23, 25, 28, 29, 32, 34, 60, 80, 83.

Solution:

3. Let $u = 3x$. This gives $du = 3dx$ and $\int e^{3x} dx = \int \frac{1}{3} e^u du = \frac{1}{3} e^u + C = \frac{1}{3} e^{3x} + C$.

5. Let $u = -0.2t$. This gives $du = -0.2dt$ and $\int 25e^{-0.2t} dt = \int \frac{25}{-0.2} e^u du = -125e^u + C = -125e^{-0.2t} + C$.

6. Let $u = t^2$. This gives $du = 2tdt$ and $\int t\cos(t^2)dt = \int \frac{1}{2}\cos u du = \frac{1}{2}\sin u + C = \frac{1}{2}\sin(t^2) + C$.

7. Let $u = 2x$. This gives $du = 2dx$ and $\int \sin(2x)dx = \int \frac{1}{2}\sin u du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(2x) + C$.

10. Let $u = y^2 + 5$. This gives $du = 2ydy$ and $\int y(y^2+5)^8 dy = \int \frac{1}{2}u^8 du = \frac{1}{18}u^9 + C = \frac{1}{18}(y^2+5)^9 + C$.

12. Let $u = 1 + 2x^3$. This gives $du = 6x^2dx$ and $\int x^2(1+2x^3)^2 dx = \int \frac{1}{6}u^2 du = \frac{1}{18}u^3 + C = \frac{1}{18}(1+2x^3)^3 + C$.

16. Let $u = 2t - 7$. This gives $du = 2dt$ and $\int (2t-7)^{73} dt = \int \frac{1}{2}u^{73} du = \frac{1}{148}u^{74} + C = \frac{1}{148}(2t-7)^{74} + C$.

18. Let $u = 4 - x$. This gives $du = -dx$ and $\int \frac{1}{\sqrt{4-x}}dx = -\int \frac{1}{\sqrt{u}}du = -2\sqrt{u} + C = -2\sqrt{4-x} + C$.

20. Let $u = x^3+1$. This gives $du = 3x^2dx$ and $\int x^2 e^{x^3+1}dx = \int \frac{1}{3}e^u du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3+1} + C$.

21. Let $u = \cos\theta + 5$. This gives $du = -\sin\theta d\theta$ and $\int \sin\theta(\cos\theta+5)^7 d\theta = -\int u^7 du = -\frac{1}{8}u^8 + C = -\frac{1}{8}(\cos\theta+5)^8 + C$.

23. Let $u = \sin\theta$. This gives $du = \cos\theta d\theta$ and $\int \sin^6\theta\cos\theta d\theta = \int u^6 du = \frac{1}{7}u^7 + C = \frac{\sin^7\theta}{7} + C$.

25. Let $u = \sin 5\theta$. This gives $du = 5\cos 5\theta d\theta$ and $\int \sin^6(5\theta)\cos(5\theta)d\theta = \int \frac{1}{5}u^6 du = \frac{1}{35}u^7 + C = \frac{\sin^7 5\theta}{35} + C$.

28. Let $u = e^t + t$. This gives $du = (e^t+1)dt$ and $\int \frac{e^t+1}{e^t+t}dt = \int \frac{1}{u}du = \ln|u| + C = \ln|e^t+t| + C$.

29. Let $u = y^2 + 4$. This gives $du = 2ydy$ and $\int \frac{y}{y^2+4}dy = \int \frac{1}{2u}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(y^2+4) + C$. (We can drop the absolute value signs since $y^2 + 4 \geq 0$ for all y.)

32. Let $u = x + e^x$. This gives $du = (1+e^x)dx$ and $\int \frac{1+e^x}{\sqrt{x+e^x}}dx = \int \frac{1}{\sqrt{u}}du = 2\sqrt{u} + C = 2\sqrt{x+e^x} + C$.

34. Let $u = x^2 + 2x + 19$. This gives $du = 2(x+1)dx$ and $\int \frac{x+1}{x^2+2x+19}dx = \int \frac{1}{2u}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x+19| + C$.

60. Let $u = 1 + x^2$. This gives $du = 2xdx$ and
$\int_{x=0}^{x=2} \frac{x}{(1+x^2)^2}dx = \int_{u=1}^{u=5} \frac{1}{2u^2}du = \left[-\frac{1}{2u}\right]_1^5 = \frac{2}{5}$.

80. The area is given by $\int_0^2 xe^{x^2}dx$. To evaluate let $u = x^2$. This gives $du = 2xdx$ and $\int_0^2 xe^{x^2}dx = \int_0^4 \frac{1}{2}e^u du = \left[\frac{1}{2}e^u\right]_0^4 = \frac{1}{2}(e^4 - 1)$.

83. (a) $\int 4x(x^2+1)dx = \int (4x^3+4x)dx = x^4 + 2x^2 + C$.
(b) With $w = x^2 + 1$, $dw = 2xdx$ and $\int 4x(x^2+1)dx = \int 2wdw = w^2 + D = (x^2+1)^2 + D$.
(c) The expressions from parts (a) and (b) look different, but they are both correct. Note that $(x^2+1)^2 + D = x^4 + 2x^2 + 1 + D$. The expressions from parts (a) and (b) differ only by a constant, so they are both correct. In effect the C from part (a) corresponds to the $1 + D$ of part (b).