The University of Sydney

MATH1902 Linear Algebra (Advanced)

Semester 1	**Assignment 1**	2009

INSTRUCTIONS

This assignment is due by 4:00pm on **Monday, 6 April, 2009**. It should be posted in the glass-fronted collection boxes on the verandah of Carslaw Level 3. These boxes are at the end of the verandah closest to Eastern Avenue. (NOT the glass-fronted collection boxes near the pyramids on Carslaw Level 3, nor the open wooden pigeonholes.) Please do not post your assignment before the due date since the boxes are also used for the collection of assignments in other units. Your assignment must be stapled inside a manilla folder, and a cover sheet must be signed and attached. The cover sheet may be downloaded from the MATH1902 website.

This assignment is worth 5% of your assessment for MATH1902. There are *three* questions. Marks allocated for each question are indicated on this sheet. You must show all working in order to receive full marks.

QUESTIONS

1. Consider the following list of vector "identities", some of which are true, some false, and others meaningless. Identify which is which. If an identity is meaningless, explain why it is so. If an identity is false, give specific values of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ which demonstrate its falsity. If an identity is true, then prove it, using either the algebraic or geometric definition of the dot and/or cross products.

   (a) $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$

   (b) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

   (c) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

   (d) $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

   (e) $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ together imply $\mathbf{b} = \mathbf{c}$

   (f) $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ together imply $\mathbf{b} = \mathbf{c}$

   (g) $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ together imply $\mathbf{b} = \mathbf{c}$

   (h) $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2$

(Question 1 is worth 10 marks)

2. A *median* of a triangle is a line that connects a vertex to the midpoint of its opposite side. Show that the medians of any triangle intersect at a point. Also show that this point divides each median in the ratio 2 : 1 (measured from the vertex to the midpoint of the opposite side).

(Question 2 is worth 10 marks)

3. In this question we will derive a formula for the (shortest) distance between two non-parallel, non-intersecting lines.

    (a) Suppose that $\mathscr{P}_1$ and $\mathscr{P}_2$ are parallel planes. Let $\mathbf{w}$ be a vector perpendicular to both planes, and suppose that A and B are points on $\mathscr{P}_1$ and $\mathscr{P}_2$ respectively. Show that the distance between the two planes is given by

    $$\frac{|\overrightarrow{AB} \cdot \mathbf{w}|}{|\mathbf{w}|}.$$

    (The distance between the planes is defined to be the minimum distance between a point from $\mathscr{P}_1$ and a point from $\mathscr{P}_2$.)

    (b) Let $\mathscr{L}_1$ and $\mathscr{L}_2$ be non-parallel, non-intersecting lines in space. Suppose that A and B are points on $\mathscr{L}_1$ and $\mathscr{L}_2$ (respectively) and that $\mathbf{u}$ and $\mathbf{v}$ are vectors parallel to $\mathscr{L}_1$ and $\mathscr{L}_2$ (respectively).

    (i) Show that there exist points P and Q on $\mathscr{L}_1$ and $\mathscr{L}_2$ (respectively) such that $\overrightarrow{PQ}$ is perpendicular to both $\mathscr{L}_1$ and $\mathscr{L}_2$.

    (ii) Show that the distance between $\mathscr{L}_1$ and $\mathscr{L}_2$ is equal to

    $$\frac{|\overrightarrow{AB} \cdot (\mathbf{u} \times \mathbf{v})|}{|\mathbf{u} \times \mathbf{v}|}.$$

(Question 3 is worth 20 marks)