THE UNIVERSITY OF SYDNEY

MATH1902 LINEAR ALGEBRA (ADVANCED)

Semester 1	Assignment 2	2009

This assignment is due on **Tuesday 2 June**, before 4 pm. Your assignment should be posted in the glass-fronted collection boxes at the western end of the verandah (closest to Eastern Avenue) on Carslaw Level 3 (not the glass-fronted collection boxes near the pyramids, nor the open wooden pigeonholes on the same level). You must download and complete an assignment cover sheet from the MATH1902 web page. Staple both the cover sheet and your assignment inside a folder. Write the initial of your family name as a large letter on the front of the folder. See page 26 of the Junior Mathematics Handbook for further guidelines regarding submission of assignments. All necessary working must be shown.

1. (10 marks). The matrix C is given by

$$C = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 1 & a & b & 3 \\ -2 & -4 & a & -6 \\ a & 4 & -a & 6 \end{bmatrix},$$

where a and b are constants.

(*i*) Calculate the determinant of C.

(*ii*) Find the values of a and b for which the matrix C is not invertible. Calculate the rank of C for each of these values.

(*iii*) Give the general solution of the system of equations $C\mathbf{x} = \mathbf{0}$ for the values of a and b for which $\operatorname{rank} C = 2$.

2. (10 marks). Let A and B be square matrices of the same size.

(*i*) Prove that the relation $(A+B)^2 = A^2 + 2AB + B^2$ implies $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$.

(*ii*) Suppose A is invertible. Decide whether the following implications hold:

(a) $(AB)^2 = A^2B^2$ implies $(AB)^3 = A^3B^3$.

(b) $(AB)^2 = A^2B^2$ implies $A(B^2A)^2B^2 = B^3A(BA)^2B$.

(c) $(AB)^2 = A^2B^2$ implies $(BA)^2 = A^2B^2$.

For each of (a), (b) and (c), provide a proof or a counterexample.