

THE UNIVERSITY OF SYDNEY  
MATH1902 Linear Algebra

---

Practice questions for Quiz 2

---

1. Which option correctly describes the following matrices?

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

- (a) Only  $A$  and  $B$  are in reduced row echelon form.  
(b) Only  $C$  is in reduced row echelon form.  
(c) Only  $B$  is in reduced row echelon form.  
(d) Only  $A$  is in reduced row echelon form.
2. Which row operations on  $A$  correspond to the matrix product  $E_3E_2E_1A$ , where  $E_1, E_2, E_3$  are the elementary matrices below?

$$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- (a)  $R_1 \leftrightarrow R_2$ ;  $R_2 := R_2 + 2R_1$ ;  $R_3 := R_3 - 3R_1$ .  
(b)  $R_1 \leftrightarrow R_2$ ;  $R_1 := R_1 - 2R_2$ ;  $R_2 := R_2 + 3R_3$ .  
(c)  $R_1 \leftrightarrow R_2$ ;  $R_1 := R_1 + 2R_2$ ;  $R_2 := R_2 + 3R_1$ .  
(d)  $R_2 \leftrightarrow R_1$ ;  $R_1 := R_1 + 2R_3$ ;  $R_2 := R_2 + 3R_3$ .
3. Which row operations convert  $\begin{bmatrix} 0 & 3 & 0 \\ 2 & 0 & -6 \\ 1 & 0 & -2 \end{bmatrix}$  to reduced row echelon form?

- (a)  $R_1 \leftrightarrow R_2$ ;  $R_1 := \frac{1}{2}R_1$ ;  $R_3 := R_3 - R_1$ ;  $R_1 := R_1 + 3R_3$ ;  $R_2 := 3R_2$ .  
(b)  $R_1 \leftrightarrow R_3$ ;  $R_2 := R_2 - 2R_1$ ;  $R_1 := R_1 - R_2$ ;  $R_3 := \frac{1}{3}R_3$ ;  $R_2 := \frac{1}{2}R_2$ ;  $R_2 \leftrightarrow R_3$ .  
(c)  $R_1 \leftrightarrow R_2$ ;  $R_1 := \frac{1}{2}R_1$ ;  $R_3 := R_3 - R_1$ ;  $R_2 := \frac{1}{3}R_2$ ;  $R_1 := R_1 - 3R_3$ .  
(d)  $R_1 \leftrightarrow R_3$ ;  $R_2 := R_2 - 2R_1$ ;  $R_1 := R_1 - R_2$ ;  $R_2 \leftrightarrow R_3$ ;  $R_2 := \frac{1}{3}R_2$ ;  $R_3 := -\frac{1}{2}R_3$ .
4. Given  $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$ , which of the following options is true?

- (a) There are matrices  $X$  and  $Y$  such that  $XA = I_2$  and  $AY = I_3$ .  
(b) There is a matrix  $Y$  such that  $AY = I_3$  but no  $X$  such that  $XA = I_2$ .  
(c) There is a matrix  $X$  such that  $XA = I_2$  but no  $Y$  such that  $AY = I_3$ .  
(d) None of the above.

5. If  $x$  is a  $1 \times 3$  row vector and  $y$  is a  $3 \times 1$  column vector, and  $yx = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ , then possible choices for  $x$  and  $y$  are

(a)  $x = [1, 0, 0]$ ,  $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ .      (b)  $x = [1, 2, 3]$ ,  $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ .

(c)  $x = [1, 1, 1]$ ,  $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ .      (d)  $x = [3, 2, 1]$ ,  $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ .

6. The inverse of  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix}$  is

(a)  $\begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ ,      (b)  $\begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$ ,      (c)  $\begin{bmatrix} 0 & -1 & -1 \\ 2 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

(d) None of the above.

7. Calculate the inverse of the matrix  $\begin{bmatrix} 5 & 3 & 13 \\ 2 & 1 & 3 \\ 5 & 2 & 1 \end{bmatrix}$ .

8. Given  $A = \begin{bmatrix} \alpha & 0 & \alpha \\ 1 & \alpha & 0 \\ 1 & 1 & 0 \end{bmatrix}$ , where  $\alpha$  is a real number, which of the following options is true?

- (a)  $A$  is invertible if  $\alpha \neq 0$  and  $\alpha \neq 1$ .  
 (b)  $A$  is invertible if  $\alpha \neq 0$ .  
 (c)  $A$  is invertible if  $\alpha \neq 1$ .  
 (d) None of the above.

9. Solve the system of equations

$$\begin{cases} 2x + 3y - z + w = -4 \\ -x - 2y = 0 \\ 3z + 2w = 0 \\ 2y + 4z + w = 3. \end{cases}$$

10. Suppose that  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & \beta & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  satisfy  $AB = BA$ . Which of the following options is true?

- (a)  $a = 0$ ,  $b = 0$ ,  $c = 0$ ,  $d = 0$ ,  $f = 0$ ,  $g = 0$ ,  $h = 0$ .  
 (b)  $a = 0$ ,  $d = 0$ ,  $f = 0$ ,  $g = 0$ ,  $h = 0$ .  
 (c)  $c = 0$ ,  $d = 0$ ,  $f = 0$ ,  $g = 0$ ,  $h = 0$ .  
 (d)  $c = 0$ ,  $d = 0$ ,  $e = 0$ ,  $f = 0$ ,  $g = 0$ .