THE UNIVERSITY OF SYDNEY

MATH1902 Linear Algebra (advanced)

Exercises for Week 2 (beginning March 9) Semester 1 2009

Reading:

Chapter 2 of the Vectors book.

Objectives:

By the end of Week 2, to achieve at least a pass level, you should

2A: be able to find the Cartesian coordinate form of position vectors in two and three dimensions.

2B: be able to find the polar form of a vector given in Cartesian form.

2C: be able to find the Cartesian form of a vector given in polar form.

2D: be able to find the coordinates of the point that divides a line segment in a given ratio.

To achieve higher than a pass level you should

2E: be able to translate problems expressed in words into vector notation.

2F: be able to reason logically with vectors in order to prove theorems in geometry.

2G: practice visualising vector figures in two and three dimensions.

2H: be able to apply the concept of linear independence to collections of vectors in two and three dimensions.

Preparatory questions. (Answers are on the next page.)

1. Given points A and B in space (with origin at O) with Cartesian coordinates $(2, -1, 3)$ and $(-1, 1, 0)$ respectively.

(i) Write the position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in Cartesian form.

(ii) Write $\overrightarrow{AB}$ in Cartesian form.

(iii) Find the length of $\overrightarrow{AB}$.

(iv) Find the Cartesian coordinates of the point P that divides AB in the ratio $3 : 4$.

2. Draw a diagram showing the vector $\mathbf{v} = 4(\cos \pi/3\, \mathbf{i} + \sin \pi/3\, \mathbf{j})$ and find the coordinates of the point P such that $\overrightarrow{OP} = \mathbf{v}$.

3. Given the vectors $\mathbf{a} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{c} = 2\mathbf{j} - \mathbf{k}$, evaluate

(i) $2\mathbf{a} - \mathbf{b} - 3\mathbf{c}$ (ii) $|\mathbf{a}|$ (iii) $\hat{\mathbf{a}}$.

Self-assessment checklist

Tick the box or boxes and seek help from your tutor, if required.

☐ I was unable to complete the Preparatory Questions.

☐ I completed the Preparatory Questions:

☐ with ease. ☐ with some effort. ☐ with difficulty.

Practice questions

4. If $\mathbf{A} = -12\mathbf{i} + 4\mathbf{j}$, find A, $\hat{\mathbf{A}}$ and the polar form of $\mathbf{A}$. (To find the polar form find r and θ and express $\mathbf{A}$ in the form $r(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$.)

5. Let $\mathbf{i}$ and $\mathbf{j}$ denote displacements of 1 km east and north, respectively. An aircraft flies 300 km southeast and then 150 km in the direction 30° west of north. Find

(i) the above displacements and their vector sum in terms of $\mathbf{i}$ and $\mathbf{j}$.

(ii) the final distance and direction of the aircraft from its starting position.

6. Suppose that no two of the points A, B, C are coincident. Show that these three points will be collinear if and only if there exist three non–zero real numbers α, β, γ such that

$$\alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c} = \mathbf{0}, \quad \alpha + \beta + \gamma = 0. \tag{1}$$

where $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the position vectors of A, B and C respectively.

7. (i) Suppose that no three of the points A, B, C, D are collinear. Show that these four points will be coplanar (lie in a plane) if and only if there exist four nonzero real numbers α, β, γ, δ, such that

$$\alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c} + \delta\mathbf{d} = \mathbf{0}, \quad \alpha + \beta + \gamma + \delta = 0. \tag{2}$$

where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ are the position vectors of A, B, C and D respectively.

(ii) Using the result of part (i), show if D is a point in the plane of the triangle ABC, and the lines AD, BD, CD cut the sides opposite A, B, C in the points R, S, T, then R divides BC in the ratio $\gamma : \beta$. Find the corresponding ratios in which S and T cut CA and AB.

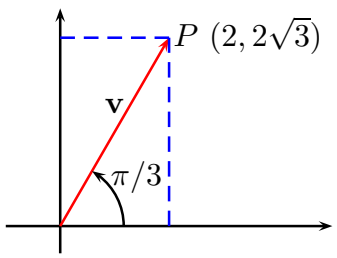
(iii) Hence show that the product of the ratios in which R, S and T divide BC, CA and AB is 1. (Theorem of Ceva.)

8. Find the sum of the three vectors determined by the diagonals of three adjacent faces of a cube passing through a given corner; the vectors being directed away from the corner. (Use Cartesian coordinates. Take the origin to be the given corner, axes along the edges of the cube, and take the edge length to be λ.)

Answers to Preparatory Questions

1. (i) $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = -\mathbf{i} + \mathbf{j}$. ($ii$) $\overrightarrow{AB} = -3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.
(iii) $|\overrightarrow{AB}| = \sqrt{22}$. ($iv$) $\overrightarrow{OP} = \frac{1}{7}(5\mathbf{i} - \mathbf{j} + 12\mathbf{k})$.

2.



3. (i) $-7\mathbf{i} - \mathbf{j} - \mathbf{k}$ (ii) 3 (iii) $-\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$

Self-assessment checklist:

Think about the work you have completed and how it relates to the objectives on the first page. This is aimed at helping you focus on how well you are going and on the areas in which you may need to do further practice or seek assistance.

In the following table, each row corresponds to one of the objectives listed on the first page. Tick the box corresponding to the level of understanding you believe you have achieved.

My understanding is:	Nil	Small	Good	Very Good	Complete
Objective 2A	☐	☐	☐	☐	☐
Objective 2B	☐	☐	☐	☐	☐
Objective 2C	☐	☐	☐	☐	☐
Objective 2D	☐	☐	☐	☐	☐
Objective 2E	☐	☐	☐	☐	☐
Objective 2F	☐	☐	☐	☐	☐
Objective 2G	☐	☐	☐	☐	☐
Objective 2H	☐	☐	☐	☐	☐

Web Quiz

There are additional self assessment tasks on the Web. Go to the Web page at

`www.maths.usyd.edu.au/u/UG/JM/MATH1902/`

and then do the Web Quiz for Week 2.