THE UNIVERSITY OF SYDNEY

MATH1902 Linear Algebra (advanced)

Exercises for Week 4 (beginning March 23) Semester 1 2009

Preliminary Reading: Chapter 4 of the Vectors book.

Objectives:

By the end of Week 4, to achieve at least a pass level, you should be able to

4A: perform simple calculations using both scalar and vector products.

4B: use the scalar triple product to calculate the volume of a parallelepiped.

4C: recognise and convert between the parametric (vector and scalar) and the Cartesian forms of the equation of a line.

To achieve higher than a pass level you should be able to

4D: use the vector product to calculate the perpendicular distance from a point to the line through two given points.

4E: calculate the distance between two lines.

4F: use the vector representation of a line to prove theorems in geometry.

Preparatory questions. (Answers are on the next page.)

1. Verify by direct calculation that $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = -(\mathbf{v} \times \mathbf{u}) \cdot \mathbf{w}$ where $\mathbf{u} = \mathbf{i} + \mathbf{j}$, $\mathbf{v} = \mathbf{j} + 2\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

2. Find the volume of the parallelepiped having $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ of Question 1 as adjacent edges.

3. Given the line ℓ with parametric equation $\mathbf{r} = \mathbf{i} + \mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$:

(i) find a vector parallel to the line ℓ;

(ii) if $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, express x, y and z in terms of t.

Self-assessment checklist

Tick the box or boxes and seek help from your tutor, if required.

☐ I was unable to complete the Preparatory Questions.

☐ I completed the Preparatory Questions:
☐ with ease. ☐ with some effort. ☐ with difficulty.

Practice questions

4. Given a point O as origin, orthonormal basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ and vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{c} = -\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, let A, B and C be the points in space such that $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$ and $\mathbf{c} = \overrightarrow{OC}$.

(i) Find a vector perpendicular to the plane containing A, B and C.

(ii) Find the perpendicular distance from A to the line through B and C.

(iii) Find the area of the triangle ABC.

(*iv*) Show that $(\mathbf{a}\times\mathbf{b})+(\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})$ is perpendicular to the plane of ABC. How is this vector related to the one you found in (*i*) and to your answer to (*iii*)?

5. (Class discussion) If $\mathbf{u}\times\mathbf{v}=\mathbf{0}$ and $\mathbf{u}\cdot\mathbf{v}=0$ is it necessary that $\mathbf{u}=0$ or $\mathbf{v}=0$?

6. (Class discussion) Given a vector $\mathbf{u}$, describe the points whose position vector $\mathbf{r}$ satisfies $\mathbf{r}\cdot(\mathbf{r}-\mathbf{u})=0$.

7. Given the points $A\,(1,-1,6)$, $B\,(2,1,0)$, $C\,(-3,2,-4)$ and $D\,(-9,1,-2)$,

(*i*) find the equation of the line through A that is parallel to BC, (a) in parametric vector form, and (b) in Cartesian form, and

(*ii*) show that D lies on this line.

8. (*i*) Find a vector that is perpendicular to both the line through the points $A\,(1,-2,-1)$ and $B\,(4,0,-3)$ and to the line ℓ whose vector equation is $\mathbf{r}=\mathbf{i}+2\mathbf{j}-\mathbf{k}+t(\mathbf{i}-6\mathbf{j}-4\mathbf{k})$.

(*ii*) Find a point L on AB and a point M on the line ℓ such that $\overrightarrow{LM}$ is perpendicular to both of these lines.

[Hint: Let L be the point on AB such that $\overrightarrow{AL}=\alpha\overrightarrow{AB}$, and M be the point with position vector $\mathbf{m}=\mathbf{i}+2\mathbf{j}-\mathbf{k}+\beta(\mathbf{i}-6\mathbf{j}-4\mathbf{k})$, where α and β are to be determined by the condition that $\overrightarrow{LM}$ is perpendicular to both lines, and therefore parallel to the vector determined in part (*i*).]

(*iii*) Show that the shortest distance between the two lines is 4/3. [Hint: $|\overrightarrow{LM}|$]. (Can you think of a way to calculate the shortest distance without finding the locations of L and M?

Answers to Preparatory Questions

2. 1

3. (*i*) $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$. (*ii*) $x=1+t$, $y=2t$, $z=1+3t$.

Self-assessment checklist:

Think about the work you have completed and how it relates to the objectives on the first page. This is aimed at helping you focus on how well you are going and on the areas in which you may need to do further practice or seek assistance.

In the following table, each row corresponds to one of the objectives listed on the first page. Tick the box corresponding to the level of understanding you believe you have achieved.

My understanding is:	Nil	Small	Good	Very Good	Complete
Objective 4A	☐	☐	☐	☐	☐
Objective 4B	☐	☐	☐	☐	☐
Objective 4C	☐	☐	☐	☐	☐
Objective 4D	☐	☐	☐	☐	☐
Objective 4E	☐	☐	☐	☐	☐
Objective 4F	☐	☐	☐	☐	☐

Web Quiz

There are additional self assessment tasks on the Web. Go to the Web page at

`www.maths.usyd.edu.au/u/UG/JM/MATH1902/`

and then do the Web Quiz for Week 4.