

Preliminary Reading:

Chapter 4 of the Vectors book.

Objectives:

By the end of Week 5, to achieve at least a pass level, you should be able to

5A: find the equation of the line through two given points,

5B: find the equation of the plane through a given point and perpendicular to a given vector,

5C: find the equation of the plane through three given non-collinear points.

To achieve higher than a pass level you should be able to

5D: use vector methods, including scalar and vector products to solve problems in geometry involving lines, planes, and other geometric figures.

Preparatory questions. (Answers are on the next page.)

1. Find the equation of the line through the points $A(1, 2, 3)$ and $B(-3, 0, 4)$ in both vector and Cartesian form.
2. Determine whether the points $M(1, 1, 1)$ and $N(5, 4, 2)$ are on the line through A and B above.
3. Find the Cartesian equation of the plane through the point $(1, 2, 0)$ and perpendicular to $2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$.

Self-assessment checklist

Tick the box or boxes and seek help from your tutor, if required.

I was unable to complete the Preparatory Questions.

I completed the Preparatory Questions:

with ease.

with some effort.

with difficulty.

Practice questions

4. Find, in Cartesian form, the equation of the plane which passes through the point $A(2, -1, 1)$ and which is perpendicular to the vector from the origin to $B(-1, 1, 3)$.
5. Find the equation of the plane containing the points $P(2, 1, -3)$, $Q(4, -1, 2)$ and $R(3, 0, 1)$.
6. Find the intersection of the line joining the points $A(1, -2, -1)$ and $B(2, 3, 1)$ with the plane passing through the points $P(2, 1, -3)$, $Q(4, -1, 2)$ and $R(3, 0, 1)$.
7. (i) Show that the vector equation of the plane passing through the three points $A(-1, 4, 2)$, $B(2, 4, 1)$ and $C(-1, 0, 1)$ is

$$(\mathbf{r} + \mathbf{i} - \mathbf{k}) \cdot (-4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k}) = 0.$$

(ii) Hence find the shortest distance from the point $D(1, -2, 1)$ to the plane.
 [Hint: Consider the component of \vec{DA} in the direction of the normal to the plane. Ans: 14/13].

8. The point P with position vector $\mathbf{r} = \vec{OP}$ lies on the sphere with centre C and radius a if and only if $|\mathbf{r} - \mathbf{c}| = a$, where $\mathbf{c} = \vec{OC}$. (Thus $|\mathbf{r} - \mathbf{c}| = a$ is the equation of the sphere.)

(i) Show that the equation of the sphere can be written in the alternative form $|\mathbf{r}|^2 - 2\mathbf{c} \cdot \mathbf{r} + |\mathbf{c}|^2 - a^2 = 0$.

(ii) Given a point B on the sphere with position vector $\mathbf{b} = \vec{OB}$ show that the line $\mathbf{r} = \mathbf{b} + t\mathbf{d}$ through B in the direction \mathbf{d} meets the sphere where $t = 0$ and where $t = -2\mathbf{d} \cdot (\mathbf{b} - \mathbf{c})/|\mathbf{d}|^2$.

(iii) Deduce from the previous part of the question that the line is tangent to the sphere if and only if $\mathbf{d} \cdot (\mathbf{b} - \mathbf{c}) = 0$.

(iv) What is the vector equation of the tangent plane to the sphere at B ?

Answers to Preparatory Questions

1. The vector parametric equation of the line is $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + t(-4\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and the Cartesian form is

$$\frac{x - 1}{-4} = \frac{y - 2}{-2} = \frac{z - 3}{1}.$$

2. Substituting the coordinates of M and N into the previous equations shows that N is on the line but M is not.

3. $2x - 5y + z = -8$.

Self-assessment checklist:

Think about the work you have completed and how it relates to the objectives on the first page. This is aimed at helping you focus on how well you are going and on the areas in which you may need to do further practice or seek assistance.

In the following table, each row corresponds to one of the objectives listed on the first page. Tick the box corresponding to the level of understanding you believe you have achieved.

My understanding is:	Nil	Small	Good	Very Good	Complete
Objective 5A	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Objective 5B	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Objective 5C	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Objective 5D	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Web Quiz

There are additional self assessment tasks on the Web. Go to the Web page at

www.maths.usyd.edu.au/u/UG/JM/MATH1902/

and then do the Web Quiz for Week 5.