THE UNIVERSITY OF SYDNEY

MATH1902 Linear Algebra (advanced)

Exercises for Week 6 (beginning April 6) Semester 1 2009

Preliminary Reading:

Chapter 1 of the Linear Algebra book.

Objectives:

By the end of Week 6, to achieve at least a pass level, you should be able to

6A: solve equations using Gaussian elimination and elementary row operations,

6B: explain why the row operations you use to solve equations must be reversible.

To achieve higher than a pass level you should be able to

6C: understand the connection between solving simultaneous linear equations and finding the intersection of lines and planes in space.

Preparatory questions. (Answers are on the next page.)

1. For the following pair of linear equations, write down an elementary row operation which eliminates x from the second equation:
$$\begin{aligned} 4x - 2y \ + z &= 3 \\ -7x + 2y - 3z &= 2 \end{aligned}$$

2. Write down the *augmented matrix* for each of the following systems of linear equations:

(i)
$$\begin{aligned} x + 2y \ + z &= 7 \\ 4x \ - y + 8z &= 3 \\ -x + 3y - 3z &= 2 \end{aligned}$$

(ii)
$$\begin{aligned} -x \ + y \ + z &= 0 \\ 11x - 2y + 8z &= 0 \\ x \ + y \ - z &= 0 \end{aligned}$$

3. Given a system of linear equations, is the operation of *simultaneously* applying the row operations $R_2 := R_2 - R_1$ and $R_1 := R_1 - R_2$ reversible? If not, why not?

Self-assessment checklist

Tick the box or boxes and seek help from your tutor, if required.

☐ I was unable to complete the Preparatory Questions.

☐ I completed the Preparatory Questions:

☐ with ease. ☐ with some effort. ☐ with difficulty.

Practice questions

4. Solve the following systems of linear equations by forming the augmented coefficient matrix and performing elementary row operations.

(i)
$$\begin{aligned} x \ + y + 2z &= 8 \\ -x - 2y + 3z &= 1 \\ 3x - 7y + 4z &= 10 \end{aligned}$$

(ii)
$$\begin{aligned} 2x + 2y + 2z &= 0 \\ -2x + 5y + 2z &= 0 \\ -7x + 7y + \ z &= 0 \end{aligned}$$

(iii)
$$\begin{aligned} x \ - y + 2z - \ w &= -1 \\ 2x \ + y - 2z - 2w &= -2 \\ -x + 2y - 4z + \ w \ &= 1 \\ 3x \qquad\quad - 3w &= -3 \end{aligned}$$

(iv)
$$\begin{aligned} 2x - 3y &= -2 \\ 2x + \ y &= 1 \\ 3x + 2y &= 1 \end{aligned}$$

5. (i) Show that the lines in three dimensional space whose equations are

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$$

and

$$\frac{x-8}{7} = y-4 = \frac{z-5}{3}$$

have a point of intersection, and find it. (Do this either by solving a system of four equations in x, y and z or by using parametric equations.)

(ii) Find the equation of the plane containing the above two lines.

6. Show that if t, u and v are distinct numbers then the following simultaneous linear equations in the unknowns a, b and c have a unique solution, for any values of the constants p, q and r:

$$\begin{aligned} a + tb + t^2c &= p \\ a + ub + u^2c &= q \\ a + vb + v^2c &= r. \end{aligned}$$

Hence show that there is a graph $y = a + bx + cx^2$ passing through the points (t, p), (u, q) and (v, r).

Answers to Preparatory Questions

1. $R_2 := R_2 + \frac{7}{4}R_1$

2. (i) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 4 & -1 & 8 & 3 \\ -1 & 3 & -3 & 2 \end{array}\right]$ (ii) $\left[\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 11 & -2 & 8 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right]$

3. In the new matrix, the second row will be the negative of the first row and in general information will be lost. For example, $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ would become $\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$ and all subsequent row operations would leave the matrix with 0's in the second column.

Self-assessment checklist:

Think about the work you have completed and how it relates to the objectives on the first page. This is aimed at helping you focus on how well you are going and on the areas in which you may need to do further practice or seek assistance.

In the following table, each row corresponds to one of the objectives listed on the first page. Tick the box corresponding to the level of understanding you believe you have achieved.

My understanding is:	Nil	Small	Good	Very Good	Complete
Objective 6A	☐	☐	☐	☐	☐
Objective 6B	☐	☐	☐	☐	☐
Objective 6C	☐	☐	☐	☐	☐

Web Quiz

There are additional self assessment tasks on the Web. Go to the Web page at

`www.maths.usyd.edu.au/u/UG/JM/MATH1902/`

and then do the Web Quiz for Week 6.