

Preliminary Reading:

Chapter 2 of the Linear Algebra book.

Objectives:

By the end of Week 7, to achieve at least a pass level, you should be able to

7A: compute sums and products of matrices,

7B: compute with row and column vectors,

7C: reduce matrices to row echelon form and to reduced row echelon form.

To achieve higher than a pass level you should be able to

7D: use the “sigma notation” for summation of series,

7E: prove the basic laws of matrix algebra,

7F: find all matrices satisfying given conditions.

Preparatory questions. (Answers are on the next page.)

1. Given the matrices $C = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & 5 \\ 6 & 1 \end{bmatrix}$, find $C + D$, $3C$, CD and DC .

2. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 2 & -6 \\ 7 & 12 & 17 \end{bmatrix}$. Use elementary row operations to transform A to a **reduced** row echelon matrix.

3. Write the column vector $\begin{bmatrix} 2 + s + 3t \\ 1 - t \\ s \\ -5 \end{bmatrix}$ as a sum of the form $\mathbf{a} + s\mathbf{b} + t\mathbf{c}$, where \mathbf{a} , \mathbf{b} and \mathbf{c} are column vectors.

Self-assessment checklist

Tick the box or boxes and seek help from your tutor, if required.

I was unable to complete the Preparatory Questions.

I completed the Preparatory Questions:

with ease.

with some effort.

with difficulty.

Practice questions

4. Let $A = \begin{bmatrix} -2 & 1 & 1 \\ -2 & 0 & 1 \\ -4 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 2 \\ 4 & 1 & -2 \\ 6 & 0 & -2 \end{bmatrix}$. Compute A^3 , BA and AB .

5. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and $D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$. Suppose that $AD = DA$, and suppose also that no two of a, b, c are equal. Show that $a_{12}, a_{21}, a_{13}, a_{31}, a_{23}$ and a_{32} are all zero.
6. Prove that $(A + B)C = AC + BC$ whenever A and B are $r \times n$ matrices and C is an $n \times p$ matrix.
7. Let \mathbf{x} be a 1×3 row vector and \mathbf{y} a 3×1 column vector. Given that

$$\mathbf{y}\mathbf{x} = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 2 & -6 \\ 6 & -3 & 9 \end{bmatrix}.$$

determine \mathbf{xy} .

8. Express $\mathbf{v} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$ as a linear combination of $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$. That is, find numbers x, y and z such that $\mathbf{v} = x\mathbf{v}_1 + y\mathbf{v}_2 + z\mathbf{v}_3$.

Answers to Preparatory Questions

1. $C + D = \begin{bmatrix} 6 & 4 \\ 9 & 3 \end{bmatrix}$, $3C = \begin{bmatrix} 6 & -3 \\ 9 & 6 \end{bmatrix}$, $CD = \begin{bmatrix} 2 & 9 \\ 24 & 17 \end{bmatrix}$, and $DC = \begin{bmatrix} 23 & 6 \\ 15 & -4 \end{bmatrix}$.
2. The row operations are

$$\begin{aligned} & \begin{bmatrix} 1 & 1 & 1 \\ -2 & 2 & -6 \\ 7 & 12 & 17 \end{bmatrix} \xrightarrow{\substack{R_2 := R_2 + 2R_1 \\ R_3 := R_3 - 7R_1}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & -4 \\ 0 & 5 & 10 \end{bmatrix} \xrightarrow{R_2 := \frac{1}{4}R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 5 & 10 \end{bmatrix} \\ & \xrightarrow{R_3 := R_3 - 5R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 15 \end{bmatrix} \xrightarrow{R_3 := \frac{1}{15}R_3} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \\ & \xrightarrow{\substack{R_1 := R_1 + R_3 \\ R_2 := R_2 - R_3}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 := R_1 - R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

3.

$$\begin{bmatrix} 2 + s + 3t \\ 1 - t \\ s \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ -5 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

Self-assessment checklist:

Think about the work you have completed and how it relates to the objectives on the first page. This is aimed at helping you focus on how well you are going and on the areas in which you may need to do further practice or seek assistance.

In the following table, each row corresponds to one of the objectives listed on the first page. Tick the box corresponding to the level of understanding you believe you have achieved.

My understanding is:	Nil	Small	Good	Very Good	Complete
Objective 7A	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Objective 7B	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Objective 7C	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Objective 7D	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Objective 7E	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Objective 7F	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Web Quiz

There are additional self assessment tasks on the Web. Go to the Web page at

www.maths.usyd.edu.au/u/UG/JM/MATH1902/

and then do the Web Quiz for Week 7.