The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH1903: Integral Calculus and Modelling (Advanced) Semester 2, 2009

Lecturers: Holger Dullin and James Parkinson

This assignment is worth 5% of your overall assessment for MATH1903

The assignment is due on **Tuesday 18th August**

Please show all working, and present your arguments clearly. After all, mathematics is about communicating your ideas. This is a skill that takes time and effort to master.

Submission Instructions: Please put your assignment in the glass-fronted collection boxes on the verandah of Carslaw Level 3. These boxes are at the end of the verandah closest to Eastern Avenue – *not* the glass-fronted collection boxes near the pyramids on Carslaw Level 3, *nor* the open wooden pigeonholes.

Your assignment must be stapled inside a manilla folder, on the front of which you should write the initial of your family name as a LARGE letter. A cover sheet must be signed and attached. Please do not post your assignment before Tuesday 18th August, since the boxes are also used for the collection of assignments in other units.

1. Suppose we define the inverse tangent function by the integral **(10 marks)**

$$\tan^{-1}(x) = \int_0^x \frac{1}{1+t^2}\,dt \qquad \text{for all } x \in \mathbb{R}.$$

Prove the following statements **directly from the above definition:**

(a) If $x \in \mathbb{R}$ then $\tan^{-1}(-x) = -\tan^{-1}(x)$.

(b) If $a, b \geq 0$ and $ab < 1$ then

$$\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}\left(\frac{a+b}{1-ab}\right).$$

Hint: Use The Fundamental Theorem of Calculus to differentiate $\tan^{-1}\left(\frac{a+x}{1-ax}\right)$.

2. Let $a < b$ and let f be a bounded function on $[a, b]$. **(10 marks)**

(a) Let P and Q be partitions of $[a.b]$. Then P is a *refinement of* Q if $Q \subseteq P$. Show that if P is a refinement of Q then

$$L_P \geq L_Q \qquad \text{and} \qquad U_P \leq U_Q,$$

where U_P is the upper Riemann sum of f using the partition P, and L_P is the lower Riemann sum of f using the partition P (similarly for U_Q and L_Q).

(b) Let P_1 and P_2 be arbitrary partitions of $[a.b]$. Show that

$$L_{P_1} \leq U_{P_2}.$$

Hint: Let P be the partition $P = P_1 \cup P_2$. Then P is a refinement of both P_1 and P_2 (called the common refinement of P_1 and P_2). Now use (a).