THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Assignment 2

MATH1903: Integral Calculus and Modelling (Advanced) — Semester 2, 2009

Lecturers: Holger Dullin and James Parkinson

This assignment is worth 5% of your overall assessment for MATH1903

The assignment is due on **Tuesday 6th October**

Please show all working, and present your arguments clearly. After all, mathematics is about communicating your ideas. This is a skill that takes time and effort to master.

Submission Instructions: Please put your assignment in the glass-fronted collection boxes on the verandah of Carslaw Level 3. These boxes are at the end of the verandah closest to Eastern Avenue – *not* the glass-fronted collection boxes near the pyramids on Carslaw Level 3, *nor* the open wooden pigeonholes.

Your assignment must be stapled inside a manilla folder, on the front of which you should write the initial of your family name as a LARGE letter. A cover sheet must be signed and attached. Please do not post your assignment before Tuesday 6th October, since the boxes are also used for the collection of assignments in other units.

1. [*2 marks*] Find a function $y(x)$ whose derivative is equal to its cosine squared and whose graph goes through the origin.

2. [*7 marks*] Consider the differential equation $\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\sqrt{|tx|}\,.$

(a) [*2 marks*] Sketch the direction field for $t \in [0, 3/2]$ and $x \in [-1/4, 1/4]$.

(b) [*2 marks*] Solve the differential equation to find the particular solution with $x(0) = 1/9$. Sketch this solution on your direction field.

(c) [*1 mark*] Compute the time t it takes this solution to reach the equilibrium.

(d) [*1 mark*] Find two solutions that start with the initial condition $x(1) = 0$.

(e) [*1 mark*] Explain why the uniqueness theorem does not apply.

3. [*11 marks*] Modelling the number of fish $P(t)$ in a lake can be based on the logistic equation with an additional constant term $-c$ that describes the number of fish caught per unit time

$$\frac{\mathrm{d}P}{\mathrm{d}t} = P(20 - P) - c\,.$$

(a) [*1 mark*] Sketch the direction field in the positive quadrant for $c = 91$.

(b) [*2 marks*] For general $c > 0$ find the equilibrium solutions. Depending on c there are either 0, 1, or 2 equilibria. For each case find the stability of the equilibria by considering the sign of $\mathrm{d}P/\mathrm{d}t$ nearby.

(c) [*8 marks*] Find the solution $P(t)$ with $P(0) = P_0 > 0$ in the two cases (i) $c = 75$ and (ii) $c > 100$. In each case either find the time τ after which the population is extinct, $P(\tau) = 0$, or find the limit of $P(t)$ as $t \to \infty$ (consider all $P_0 > 0$).