

Sample QUIZ 2, MATH1903

1. Give an upper bound for the error in approximating $\ln 2$ by $T_3(1)$, the Taylor polynomial of $\ln(1+x)$ about $x=0$ of degree 3.
2. Using Taylor series or otherwise, find the limit $\lim_{x \rightarrow 0} \frac{\sin 2x(\cos x - 1)}{1 - e^{x^3}}$.
3. Find the general solution of the differential equation $\frac{dy}{dx} = (x-1)(x-2)$.
4. Find the general solution of the differential equation $\frac{dy}{dx} = (y-1)(y-2)$.
5. Find the particular solution of the differential equation from question 4 that satisfies the initial condition $y(0) = 3$.
6. Determine whether the equilibrium solution of the differential equation $\frac{dy}{dx} = e^y - 2$ with $0 < y < 5$ is stable or not.
7. Which of the following differential equations is separable?
 - (a) $\frac{dy}{dx} = x^2 + 4y^2$
 - (b) $\frac{dy}{dx} = 1 + 3x^2 + y^2 + 3x^2y^2$
 - (c) $\frac{dy}{dx} = 6x^2y - x^2 + 2$
 - (d) $\frac{dy}{dx} = 4(\sin^2 x)y + \cos^2 x$
8. Which of the following differential equations is linear?
 - (a) $\frac{dy}{dx} = x^2 - 4y^2$
 - (b) $\frac{dy}{dx} = 1 + 4x + y^2 + 4xy^2$
 - (c) $\frac{dy}{dx} = \frac{6x^2}{y} - x^2$
 - (d) $\frac{dy}{dx} = 4(\cos^2 x)y + \sin^2 x$
9. Find the general solution of the differential equation $\frac{dy}{dx} + (x + \frac{1}{x})y = 2$
10. Given the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$ for $y(x)$ and the change of variables $z = y/x$, find the differential equation for $z(x)$.
11. Find the differential equation for the family of curves $y = Cx + x^2$
12. For which x_0 does the initial value problem $y(x_0) = 0$ for the differential equation from question 11 not have a unique solution?

Answers:

1. $|R_3(1)| \leq 1/4$

2. 1

3. $y(x) = 2x - \frac{3}{2}x^2 + \frac{1}{3}x^3 + C$

4. $y(x) = \frac{2 - Ce^x}{1 - Ce^x}$

5. $y(x) = \frac{4 - e^x}{2 - e^x}$

6. unstable

7. (b)

8. (d)

9. $y(x) = \frac{2}{x} + C \frac{e^{-x^2/2}}{x}$

10. $\frac{dz}{dx} = \frac{1}{xz}$

11. $\frac{dy}{dx} = \frac{y}{x} + x$

12. $x_0 = 0$