

Sample Quiz 1

MATH1903: Integral Calculus and Modelling (Advanced)

Semester 2, 2009

Lecturers: Holger Dullin and James Parkinson

Quiz 1 covers material up to and including lecture 8. The real quiz will have 10 questions, each worth 1 mark each. There will be answer boxes for you to write your answers in, and *only* your final answers will be marked (1 mark if correct and 0 marks otherwise). Calculators are *not* permitted.

1. Find a closed formula for the lower Riemann sum of $f(x) = e^x$ over the interval $[0, 1]$ using the partition P of $[0, 1]$ into n equal parts.
2. Compute the upper Riemann sum of $f(x) = \sin \pi x$ over the interval $[0, 3/2]$ using the partition $P = \{0, 1/6, 1, 5/4, 3/2\}$.
3. Given that $f(x) = x \int_0^{2x} t e^{-t} dt$, find $f''(1)$.
4. Find the derivative of the function $f(x) = \int_{\sin x}^{3+e^x} \sin t dt$.
5. Given that $\sin(e^x) = \int_0^x e^t f(t) dt$, find $f(x)$.
6. Find the value of $x > 0$ which maximises the function $I(x) = \int_0^x (1 - t^2)(t^2 - 16) dt$.
7. Find the area between the curves $y = \sin \pi x$ and $y = \cos \pi x$ with $1/2 \leq x \leq 3/2$.
8. Find the area bounded by the curve $y = \ln x$, the x -axis, and the line $x = e$.
9. Compute the volume of the solid obtained by rotating about the x -axis the region bounded by the curve $y = \sinh x$, the x -axis, and the line $x = 2$.
10. Compute the volume of the solid obtained by rotating about the y -axis the region bounded by the curve $y = e^{-x^2}$, the x -axis, the y -axis, and the line $x = 1$.
11. Find the length of the curve with parametrisation

$$x(t) = t - \sin t \quad \text{and} \quad y(t) = 1 - \cos t \quad \text{with} \quad t \in [0, 2\pi].$$

12. Compute the length of the graph $y = \cosh x$ between $x = a$ and $x = b$.
13. Compute the surface area of the solid obtained by revolving the part of the graph of $y = \sqrt{x}$ between $x = 0$ and $x = 2$ around the x -axis. Remember to include any end caps.
14. Compute the value of the improper integral $\int_0^{\infty} e^{-x} \cos x \, dx$.
15. Decide if the improper integral $\int_0^1 \frac{1}{x^2} \sin\left(\frac{1}{x}\right) \, dx$ exists.
16. Decide if the improper integral $\int_0^1 \frac{\cosh x}{\sqrt{x}} \, dx$ exists.
17. Compute the indefinite integral $\int x^n \ln x \, dx$, where $n \neq -1$.
18. Compute the improper integral $\int_0^1 \frac{x}{\sqrt{1-x}} \, dx$.
19. Change variables to find the indefinite integral $\int \frac{x^2}{\sqrt{1+x^2}} \, dx$, expressing your final answer in terms of x .
20. Find a reduction formula for the integral $I_n = \int x^n \cos x \, dx$.