

Quiz 1

MATH1903: Integral Calculus and Modelling (Advanced)

Semester 2, 2009

Lecturers: Holger Dullin and James Parkinson

Name: Solutions

SID: _____

*Please write your answers in the answer boxes. Calculators are not permitted
Each question is worth 1 mark, and half marks are not awarded*

1. Is the following statement true or false?

“If $p \geq 1$ then $\int_1^\infty \frac{1}{x^p} dx$ exists”

Answer: True False

2. Find the length of the curve in 3 dimensional space with parametrisation

$$x(t) = 2 \sin t - 1, \quad y(t) = 2 - 2 \cos t, \quad z(t) = 2t^{3/2}, \quad \text{with } t \in [0, 1].$$

Answer: $\frac{2}{27} (13^{3/2} - 8)$

3. By evaluating an appropriate improper integral, compute the area between the curves $y = e^{-2x}$ and $y = -x^{-2}$ with $x \geq 1$.

Answer:

$$1 + \frac{1}{2e^2}$$

4. Compute the lower Riemann sum of $f(x) = x^2 - x - 2$ over the interval $[-3, 3]$ using the partition $P = \{-3, -2, -1, 2, 3\}$.

Answer:

$$-\frac{11}{4}$$

5. Given that $f(x) = x \int_0^{2x} te^{-t} dt$, find $f''(x)$.

Answer:

$$12xe^{-2x} - 8x^2e^{-2x}$$

6. Compute the integral $\int_{\frac{\pi}{4}}^{\pi} \cos^3 x dx$.

Answer:

$$-\frac{5}{6\sqrt{2}}$$

7. Decide if the integral $\int_0^{\infty} (\cosh x - 4 \sin x) e^{-x} dx$ exists.

Answer: Exists / ~~Does not exist~~

8. Compute the value of the improper integral $\int_1^{\infty} \frac{(\ln x)^2}{x^2} dx$.

Answer:

2

9. Find the volume of the solid obtained by revolving about the y -axis the region bounded by the curve $y = \sin x$, the x -axis, the line $x = \pi/2$, and the line $x = \pi$.

Answer:

$$2\pi^2 - 2\pi$$

10. Find a closed formula for the lower Riemann sum of $f(x) = 2^x$ over the interval $[0, 3]$ using the partition of $[0, 3]$ into n equal parts.

Answer:

$$\frac{3}{n} \cdot \frac{1 - 2^3}{1 - 2^{3/n}}$$