

Quiz 1

MATH1903: Integral Calculus and Modelling (Advanced)

Semester 2, 2009

Lecturers: Holger Dullin and James Parkinson

Name: Solutions

SID: \_\_\_\_\_

*Please write your answers in the answer boxes. Calculators are not permitted  
Each question is worth 1 mark, and half marks are not awarded*

1. Is the following statement true or false?

“If  $p < 1$  then  $\int_0^1 \frac{1}{x^p} dx$  exists”

Answer:  True  False

2. Given that  $f(x) = x \int_0^{3x} t \cos t dt$ , find  $f''(x)$ .

Answer:

$$27x \cos(3x) - 27x^2 \sin(3x)$$

3. Find the length of the curve in 3 dimensional space with parametrisation

$$x(t) = \sin t + 1, \quad y(t) = 1 - \cos t, \quad z(t) = 2t^{3/2}, \quad \text{with } t \in [0, 1].$$

Answer:

$$\frac{2}{27} (10^{3/2} - 1)$$

4. Compute the lower Riemann sum of  $f(x) = x^2 - 4x + 3$  over the interval  $[-2, 4]$  using the partition  $P = \{-2, 0, 1, 3, 4\}$ .

Answer:

4

5. Compute the integral  $\int_{\frac{\pi}{4}}^{\pi} \sin^3 x \, dx$ .

Answer:

$$\frac{2}{3} + \frac{5\sqrt{2}}{12}$$

6. By evaluating an appropriate improper integral, compute the area between the curves  $y = x^{-3}$  and  $y = -e^{-x}$  with  $x \geq 1$ .

Answer:

$$\frac{1}{2} + \frac{1}{e}$$

7. Decide if the integral  $\int_0^{\infty} (2 \sin x - \cosh x) e^{-x} dx$  exists.

Answer:    Exists / ~~Does not exist~~

8. Find a closed formula for the lower Riemann sum of  $f(x) = 3^x$  over the interval  $[0, 2]$  using the partition of  $[0, 2]$  into  $n$  equal parts.

Answer:     $\frac{2}{n} \cdot \frac{1-9}{1-3^{2/n}}$

9. Find the volume of the solid obtained by revolving about the  $y$ -axis the region bounded by the curve  $y = \ln x$ , the  $x$ -axis, and the line  $x = e$ .

Answer:  $\frac{\pi(1+e^2)}{2}$

10. Compute the value of the improper integral  $\int_1^{\infty} \frac{(\ln x)^2}{x^2} dx$ .

Answer: 2