

Quiz 1

MATH1903: Integral Calculus and Modelling (Advanced)

Semester 2, 2009

Lecturers: Holger Dullin and James Parkinson

Name: Solutions

SID: \_\_\_\_\_

*Please write your answers in the answer boxes. Calculators are not permitted  
Each question is worth 1 mark, and half marks are not awarded*

1. Is the following statement true or false?

"If  $p \leq 1$  then  $\int_0^1 \frac{1}{x^p} dx$  exists"

Answer: True / ~~False~~

2. Compute the integral  $\int_0^{\frac{3\pi}{4}} \cos^3 x dx$ .

Answer:  $\frac{5\sqrt{2}}{12}$

3. Given that  $f(x) = x \int_0^{5x} t \sinh t dt$ , find  $f''(x)$ .

Answer:

$$75x \sinh(5x) + 125x^2 \cosh(5x)$$

4. Compute the lower Riemann sum of  $f(x) = x^2 + x - 2$  over the interval  $[-4, 2]$  using the partition  $P = \{-4, -3, -1, 1, 2\}$ .

Answer:

$$-\frac{9}{2}$$

5. By evaluating an appropriate improper integral, compute the area between the curves  $y = x^{-2}$  and  $y = -e^{-4x}$  with  $x \geq 1$ .

Answer:

$$1 + \frac{1}{4e^4}$$

6. Decide if the integral  $\int_0^{\infty} (\cos x + 3 \sinh x) e^{-x} dx$  exists.

Answer:    Exists ~~Does not exist~~

7. Find the length of the curve in 3 dimensional space with parametrisation

$$x(t) = 3 \sin t - 4, \quad y(t) = 3 \cos t + 7, \quad z(t) = 2t^{3/2}, \quad \text{with } t \in [0, 1].$$

Answer:

$$4\sqrt{2} - 2$$

8: Compute the value of the improper integral  $\int_1^{\infty} \frac{(\ln x)^2}{x^3} dx$ .

Answer:

$$\frac{1}{4}$$

9. Find a closed formula for the lower Riemann sum of  $f(x) = 5^x$  over the interval  $[0, 3]$  using the partition of  $[0, 3]$  into  $n$  equal parts.

Answer:  $\frac{3}{n} \cdot \frac{1-5^3}{1-5^{3/n}}$

10. Find the volume of the solid obtained by revolving about the  $y$ -axis the region bounded by the curve  $y = e^x$ , the  $x$ -axis, the  $y$ -axis, and the line  $x = 1$ .

Answer:  $2\pi$