1. Write down an algorithm which, given a balanced string of brackets produces a ‘planar diagram’. Then, given the following balanced strings of brackets, use the algorithm to produce the corresponding planar diagrams:

   (i)  ( )( )( )
   (ii) ((( ))( )
   (iii) ((( )))
   (iv) ((( )( )(( ))( ))).

2. Construct balanced strings of brackets corresponding to the following planar diagrams

   (i)
   (ii)
   (iii)

3. For the planar diagrams you found in question 1

   (i) construct the corresponding balanced strings of brackets, and
   (ii) construct the corresponding standard tableaux.

4. Given $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots \}$.

   Let $A = \{ n \in \mathbb{Z} \mid n \text{ is divisible by } 2 \}$,
   $B = \{ n \in \mathbb{Z} \mid n \text{ is divisible by } 3 \}$ and
   $C = \{ n \in \mathbb{Z} \mid n \leq 20 \}$. Determine the following sets

   (i) $A \cap B$
   (ii) $A \cup B$
   (iii) $A \setminus B$
   (iv) $A \cap B \cap C$
   (v) $C \setminus (A \cup B)$
   (vi) $(C \setminus A) \cap (C \setminus B)$.

5. Let $X$ be the set of planar diagrams on $2n$ points. For $i = 2, 3, \ldots, 2n$ let $A_i$ be the subset of $X$ consisting of diagrams in which the point 1 is joined to $i$. Let $B_j$ ($j = 1, 3, 4, \ldots, 2n$) be the subset of $X$ in which 2 is joined to $j$.

   (i) For $n = 3$, write down the sets $A_2, A_3, A_4, A_5, A_6, B_1, B_3, B_4, B_5, B_6$.
   (ii) Find $|A_i|, |B_j|, |A_i \cap B_j|$ for $n = 3$ and all values of $i$ and $j$.
   (iii) For general $n$, determine the values of $i$ and $j$ for which $A_i \cap B_j$ is not empty.
   (iv) Show that $c_n$ (the $n$-th Catalan number) equals $\sum_{i=2}^{2n} |A_i|$.
1. Write down an algorithm which given a standard tableau of two rows of length $n$ produces a balanced string of $n$ pairs of brackets. Given the following tableaux use your algorithm to produce the balanced strings

(i) \[
\begin{array}{cccc}
1 & 3 & 5 & 7 \\
2 & 4 & 6 & 8 \\
\end{array}
\]

(ii) \[
\begin{array}{cccc}
1 & 2 & 4 & 7 & 8 \\
3 & 5 & 6 & 9 & 10 \\
\end{array}
\]

2. If $A = \{2, \{3, 4\}, 3, 4, 5, \{\}\}$ and $B = \{3, \{4\}\}$, write down $A \cap B$, $A \cup B$, $A \setminus B$ and then determine which of the following are true:

(i) $3 \in A$  \hspace{1cm} (ii) $\{3\} \in A$  \hspace{1cm} (iii) $\{3\} \subseteq A$

(iv) $\{3, 4\} \in A$  \hspace{1cm} (v) $\{3, 4\} \subseteq A$  \hspace{1cm} (vi) $A \setminus A \in A$

(vii) $4 \notin A$  \hspace{1cm} (viii) $4 \notin B$  \hspace{1cm} (ix) $4 \not\subseteq B$