1. For the following sets \( X, Y \) and function \( X \rightarrow Y \), determine whether the function is injective or surjective

(i) \( X = \mathbb{N}, Y = \mathbb{N}, f(x) = x + 1 \).

(ii) \( X = \mathbb{Z}, Y = \mathbb{Z}, g(x) = x + 1 \).

(iii) \( X = \mathbb{Z}, Y = \mathbb{Z}, h(x) = x^2 + 5 \).

(iv) \( X = \mathbb{Z}, Y = \mathbb{Z}, p(x) = x^3 + 1 \).

(v) \( X = \text{any set}, Y = \mathcal{P}(X), h(x) = \{x\} \).

2. Compute expressions for the compositions of the following functions from Question 1.

(i) \( g \circ h \) and \( h \circ g \).

Are they injective? Surjective?

3. An \((n, n)\) standard tableau is a \(2 \times n\) array of boxes into which the integers \(1, 2, \ldots, 2n\) are inserted so that the rows increase rightwards and the columns downwards (e.g.,

\[
\begin{array}{ccc}
1 & 2 & 4 \\
3 & 5 & 6 \\
\end{array}
\]

is permitted, but

\[
\begin{array}{ccc}
1 & 4 & 5 \\
2 & 3 & 6 \\
\end{array}
\]

is not permitted.)

Let \( \mathcal{T}_n \) be the set of all \((n, n)\) standard tableaux and let \( \mathcal{P}_n \) be the set of planar diagrams on \(2n\) points.

(i) Show that there is a map \( f : \mathcal{P}_n \rightarrow \mathcal{T}_n \) where, for \( D \in \mathcal{P}_n \), \( f(D) \) has second row equal to the set of right ends in \( D \).

(ii) Describe how to assign a planar diagram on \(2n\) points to an \((n, n)\) standard tableau. Hence find a function \( g : \mathcal{T}_n \rightarrow \mathcal{P}_n \) such that \( f \circ g = \text{Id}_{\mathcal{T}_n} \) and \( g \circ f = \text{Id}_{\mathcal{P}_n} \).

4. Suppose \( f : A \rightarrow B \) and \( g : B \rightarrow A \) are functions which satisfy \( g \circ f = \text{Id}_A \). Show carefully that \( g \) is surjective and \( f \) is injective.

5. Let \( f : \{1, 2, 3, 4, 5, 6\} \rightarrow \{1, 2, 3, 4, 5, 6\} \) be the permutation defined by \( f(1) = 5 \), \( f(2) = 1 \), \( f(3) = 6 \), \( f(4) = 2 \), \( f(5) = 4 \) and \( f(6) = 3 \).

(i) What is the parity of \( f \)?

(ii) Define \( f^2 = f \circ f \), \( f^3 = f \circ f \circ f \), and so on. What is the smallest integer \( k > 0 \) such that \( f^k \) is the identity?

(iii) Given any permutation \( f \) of a finite set, explain why there is always an integer \( k > 0 \) such that \( f^k \) is the identity function.
1. Let $f : X \to Y$ and $g : Y \to Z$ be functions.
   (i) Show that if $f$ and $g$ are surjective then so is $g \circ f$.
   (ii) Show that if $f$ and $g$ are injective then so is $g \circ f$.
   (iii) Deduce that $h(x) = (1 + x)^n : \mathbb{Z} \to \mathbb{Z}$ is injective if $n$ is odd, by defining $h$ as a composition of two injective functions $f, g : \mathbb{Z} \to \mathbb{Z}$.

2. Let $C_n = \{\pm 1, \pm 2, \cdots, \pm n\}$.
   (i) How many permutations (i.e., bijective maps) $f : C_n \to C_n$ are there such that $f(-x) = -f(x)$ for all $x \in C_n$?
   (ii) How many of the permutations of (i) satisfy the additional restriction that the product $f(1)f(2)\cdots f(n)$ is positive?
   (iii) Show that the set of permutations of (i) is closed under composition (i.e., if the permutations $f$ and $g$ of $C_n$ satisfy the condition given in (i), then so does $f \circ g$).