1. Let $X$ be the set of all functions from $A = \{1, 2, 3\}$ to $B = \{a, b\}$. Define two functions $f$ and $g$ in $X$ to be equivalent if there is a permutation $h$ of $A$ such that $g = f \circ h$.

(i) Draw an arrow diagram for each element of $X$.

(ii) Find the equivalence class $[f]$ for each $f \in X$.

(iii) Check the following assertions:

(a) $f \in [f]$;

(b) if $f \in [g]$, then $[f] = [g]$;

(c) if $[f] \neq [g]$, then $[f] \cap [g] = \emptyset$.

2. How many numbers $n$ are there with $1 \leq n \leq 500$ which are not divisible by 5, 6 or 14?

3. (i) Show that for positive integers $m$ and $n$,

$$\sum_{k_1 + \cdots + k_m = n} \binom{n}{k_1, k_2, \ldots, k_m} = m^n.$$  

(ii) With notation as in (i), show that

(a) $\sum_{k_1 + \cdots + k_m = n} \binom{n}{k_1, k_2, \ldots, k_m} (-1)^{k_2 + k_4 + \cdots + k_m} = 0$ if $m$ is even;

(b) $\sum_{k_1 + \cdots + k_m = n} \binom{n}{k_1, k_2, \ldots, k_m} (-1)^{k_2 + k_4 + \cdots + k_{m-1}} = 1$ if $m$ is odd.

4. By counting a suitable set in “two ways”, prove the binomial identity

$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$  

5. (i) Find a switching circuit corresponding to the following Boolean function:

<table>
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<tr>
<th>$p$</th>
<th>$q$</th>
<th>$r$</th>
<th>$f(p, q, r)$</th>
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(ii) Hence write a Boolean expression for $f(p, q, r)$.
1. (i) How many permutations of the 26 letters $abc\ldots xyz$ contain the word $cat$?

(ii) Using the inclusion-exclusion principle, or otherwise, find the number of permutations of the 26 letters $abc\ldots xyz$ which do not contain the words $the$, $cat$ or $bag$.

2. (i) Draw the table of values for the Boolean expression $xy \lor z'$.

(ii) How many Boolean functions of $n$ variables are there?

(iii) Using the rules of Boolean algebra, prove the following identities for Boolean expressions $f$ and $g$.

(a) If $f \lor g = f$ for all $f$, then $g = 0$.

(b) If $fg = f$ for all $f$, then $g = 1$.

(c) If $f \lor g = 1$ and $fg = 0$, then $g = f'$.