1. Draw up truth tables for the following propositions

   \( (i) \ p \Rightarrow (q \lor \sim r) \)
   \( (ii) \ p \land (q \lor \sim r) \)
   \( (iii) \ (p \land q) \lor \sim r \)
   \( (iv) \ \sim (p \lor q) \lor (\sim p \land q) \lor r \)

2. Let \( p \) be the statement \( 1 + 1 = 2 \) and let \( q \) be the statement \( 2 + 2 = 4 \).

   \( (i) \) Decide if the following propositions are true or false.
   
   \( (a) \ p \Rightarrow \sim q \)
   \( (b) \ p \Rightarrow q \)
   \( (c) \ \sim p \Rightarrow q \)
   \( (d) \ \sim p \Rightarrow \sim q \)

   \( (ii) \) Write English translations of each of the above statements.

3. \( (i) \) Show that the following propositions are equivalent

   \( p \Rightarrow q \quad \text{and} \quad q \lor \sim p. \)

   \( (ii) \) Hence write a Boolean expression which does not contain implication (\( \Rightarrow \)) which is equivalent to

   \[ p \Rightarrow (q \lor (\sim r \Rightarrow \sim p)). \]

4. State the converse and the contrapositive of the sentence:

   “A positive integer is a prime only if it has no divisors other than 1 and itself”

   Is the converse true?

5. Determine whether the following propositions are tautologies

   \( (i) \) \( (\sim p \land (p \Rightarrow q)) \Rightarrow \sim q. \)
   \( (ii) \) \( (\sim q \land (p \Rightarrow q)) \Rightarrow \sim p. \)

6. Dr Bill was having trouble with his old car, so he took it to his pedantic mechanic. After a careful inspection of the engine, the mechanic (knowing that Dr Bill is a mathematician who likes precise language) said: “It is hard to be sure, but either it is true that if the spark plugs and the points are good then you need a new distributor cap, or else it is true that if the points are good but you need new spark plugs then your distributor cap is good, but not both.”

   Supposing that at least one of the three parts mentioned by the mechanic needs replacing, what should Dr Bill replace?
1. Write down all 16 truth tables for two propositions $p$ and $q$. For each truth table, write down a compound proposition formed from $p$, $q \land$, $\lor$ and $\sim$ which has the given truth table.

2. (i) Decide if the following expressions are tautologies

(a) $(\sim(p \Rightarrow q)) \Rightarrow (p \lor q)$
(b) $(p \Rightarrow q) \lor (\sim p \lor q)$

(ii) Suppose the universal set $U$ is the set of all human beings and that $P(x)$ is the statement “$x$ is musical” while $Q(x)$ is the statement “$x$ is logical”. Write English statements for the following

(a) $(\forall x) (P(x) \Rightarrow Q(x))$
(b) $(\exists x) (P(x) \lor \sim Q(x))$
(c) $\sim (\exists x (P(x) \land \sim Q(x)))$

(iii) Write down the negation of the statement

$\forall x (P(x) \Rightarrow Q(x))$

both in symbolic form and as an English sentence. Here $P(x), Q(x)$ are defined in (ii).