1. (i) Given languages $A = \{0, 11\}$ and $B = \{1, 10, 110\}$ over the alphabet $\{0, 1\}$, write down $AB$ and $BA$.

(ii) Give an example of languages $A$ and $B$ over $\{0, 1\}$ such that $|AB| \neq |A||B|$.

2. Given the alphabet $\Sigma = \{a, b, c\}$, describe the formal languages designated by the following regular expressions:

(i) $a + bc$

(ii) $a + bc^*$

(iii) $a + (bc)^*$

(iv) $(ab)^*c^*$

(v) $(ab)^*(cb)^*$

(vi) $(abca)^*$

3. Write down a regular expression which designates the following languages

(i) $\{\varepsilon, b, a, a^2, a^3, a^4, \ldots \}$.

(ii) $\{\varepsilon, a, b, c, bc, bc^2c, bc^3c, \ldots \}$

(iii) All words containing $bca$ over the alphabet $\{a, b, c\}$.

4. Let $\Sigma = \{a, b, c\}$ and let $A$ be the smallest subset of $\Sigma^*$ such that (a) $\varepsilon \in A$ and (b) if $x \in A$, then $xa \in A$, $xab \in A$ and $xbc \in A$.

(i) Write out the strings of length 2 and 3 in $A$.

(ii) Which of $abaaabca$, $bcaababa$, $abbaabca$ is in $A$?

(iii) Let $x_n$ be the number of strings in $A$ of length $n$ and show that $x_n = x_{n-1} + 2x_{n-2}$.

(iv) Find $x_{100}$.

5. Prove the following relations in the algebra of regular expressions:

(i) $(r + s)^* = (r^*s)^*r^*$

(ii) $(rs)^* = \varepsilon + r(sr)^*s$
1. Let $A = \{00, 11\}$ and $B = \{01, 0\}$. Find the following sets

(i) $AB$  (ii) $BA$  (iii) $A^3$  (iv) $B^2$  (v) $A^*$  (vi) $AB^*$

2. Determine whether the string 11101 is in the language defined by the following regular expressions.

(i) $(0 + 1)^*$  (ii) $1^*0^*1^*$  (iii) $111^*01$  (iv) $(11)^*(01)^*$
(v) $(111)^*0^*1$  (vi) $(111 + 000)(00 + 11)$