1. (i) Which of the following strings are accepted by the NFA which appears above?
   (a) 001101  (b) 0001  (c) 11  
   (d) 110100  (e) 1111  (f) 00100

(ii) For each regular expression below, state whether or not it designates the language accepted by the finite state machine.
   (a) \((0 + 1)^*11(0 + 1)^*\)
   (b) \((01)^*11(0 + 1)^*\)
   (c) \((01)^*11(01)^*\)

Solution

(i) (a) Not accepted.  (b) Not accepted.  (c) Accepted.  
   (d) Accepted.  (e) Accepted.  (f) Not accepted.

(ii) (a) No. This includes the string 0011, which is not accepted.
     (b) Yes.
     (c) No. This language does not include 110, which is accepted.

2. (i) Draw the DFA that corresponds to the following transition function. The alphabet is \(\{a, b\}\), the initial state is \(A\) and the only accepting state is \(B\).

\[
\begin{array}{c|cc}
 & a & b \\
\hline
A & B & C \\
B & B & C \\
C & B & D \\
D & D & D \\
\end{array}
\]

(ii) Which of the following strings are accepted by the DFA given in (i)?
   (a) \(aaa\)  (b) \(aaab\)  (c) \(bbab\)  (d) \(baba\)

(iii) Describe the language accepted by the machine in (i).

(iv) Find a regular expression which designates the language found in (iii).
Solution

Note: the solution in the textbook is incorrect.

(i) 

![Diagram](image)

(ii) (a) Accepted  (b) Not accepted  (c) Not accepted  (d) Accepted

(iii) The language consists of all possible combinations of the strings $a$ and $ba$ except the empty string.

(iv) $(a + ba)(a + ba)^*$.  

3. Construct a DFA that accepts only those strings of lowercase letters which end in “ing”.

Solution

Let $\Sigma$ be the set of 26 lowercase letters: a, b, c, ...

![Diagram](image)

4. Given regular expressions $r_1 = ab^*$ and $r_2 = a^*b$.

(i) Find NFA’s $M_1$ and $M_2$ which accept the languages $L(r_1)$ and $L(r_2)$.

(ii) Describe how to construct new NFA’s (using $M_1$ and $M_2$) which accept the languages

(a) $L(r_1 + r_2)$,  
(b) $L(r_1 r_2)$,  
(c) $L(r_1^*)$.  


Solution

(i) \( M_1 \)

\begin{align*}
\quad & a & b \\
\rightarrow & & \\
\quad & a & b
\end{align*}

\( M_2 \)

\begin{align*}
\quad & a & b \\
\rightarrow & & \\
\quad & a & b
\end{align*}

(ii) (a) \( L(r_1 + r_2) \)

\begin{align*}
\quad & a & b \\
\rightarrow & & \\
\quad & a & b
\end{align*}

(b) \( L(r_1r_2) \)

\begin{align*}
\quad & a & b \\
\rightarrow & & \\
\quad & b & b
\end{align*}

(c) \( L(r_1^* \text{c}) \)

\begin{align*}
\quad & a & b \\
\rightarrow & & \\
\quad & a & b
\end{align*}
1. Which of the following strings are accepted by the NFA which appears above?

   (a) 001101   (b) 0001   (c) 11
   (d) 110100   (e) 1111   (f) 00100

(ii) For each regular expression, state whether or not it designates the language accepted by the finite state machine.

   (a) $0^*(0 + 1)1(0 + 1)^*$
   (b) $0^*1(00^*1)^*(0 + 1)^*$
   (c) $(0 + 10)^*11(0 + 1)^*$

Solution

(i) (a) Accepted.   (b) Not accepted.   (c) Accepted.
    (d) Accepted.   (e) Accepted.   (f) Not accepted.

(ii) (a) No. This includes the string 01, which is not accepted.
     (b) No. This includes the string 1, which is not accepted.
     (c) Yes. Both the regular expression and the machine describe the language of strings of 0’s and 1’s which contain two consecutive 1’s.

2. (i) Construct an NFA which accepts the language $L$ over the alphabet $\{0, 1\}$ where

   (a) $L$ consists of all strings which end in 010 and do not contain any other occurrences of the sequence 010.
   (b) $L$ consists of all strings which do not contain the sequence 010.

(ii) Write down a regular expression which designates the language in part (i)(a).
**Solution**

(i) (a) A possible solution is

(b) A possible solution is

(ii) A possible regular expression for this language is \((1 + 00^*11)^*00^*10\).