SUBRINGS

Definition. Let $R$ be a ring. A subset $S$ of $R$ is called a subring if it satisfies the following four conditions:

1. The zero element 0 of $R$ lies in $S$.
2. $S$ is closed under addition, i.e. $a, b \in S \Rightarrow a + b \in S$.
3. $S$ is closed under taking negatives, i.e. $a \in S \Rightarrow -a \in S$.
4. $S$ is closed under multiplication, i.e. $a, b \in S \Rightarrow ab \in S$.

Definition. Let $F$ be a field. A subfield of $F$ is a subring $S$ of $F$ which satisfies the following extra conditions:

5. The identity element 1 of $F$ lies in $S$.
6. $S$ is closed under taking inverses, i.e. $0 \neq a \in S \Rightarrow a^{-1} \in S$. 