

This assignment is due at 5pm on Thursday 31 March at room 626.

1. (i) The population N of feral cats in bushland is known to have the following intrinsic dynamics:

$$\frac{dN}{dt} = rN^2 \left(1 - \frac{N}{K}\right)$$

where K is the carrying capacity (reflecting prey availability) and r and K are both positive.

- (a) Find all steady states of this equation and determine their stability using linearised stability analysis.
- (b) Sketch the phase portrait for this equation. Does the stability of the steady states that you have found using the phase portrait agree with the linear analysis? Why or why not?

- (ii) A national park which is home to large populations of small marsupials decides to work on reducing the cat population in the park. They employ a contractor to work for a fixed number of days each month on baiting and shooting cats. Feral cats, however, also live on adjacent properties and tend to migrate into the park when the cat population within the park becomes lower.

- (a) Explain why the following equation is a suitable model for the population of feral cats in the national park when the contractor is working on their extermination:

$$\frac{dN}{dt} = rN^2 \left(1 - \frac{N}{K}\right) - EN + M(K - N).$$

Here E and M are both positive numbers. Explain what E and M represent.

- (b) On a phase portrait diagram, sketch the curve describing the intrinsic dynamics and the line $EN - M(K - N)$. Explain why there are at most three steady states.
- (c) Using the phase portrait or otherwise find the stability of each steady state.
- (d) The national park's management became very discouraged when they had only reduced feral cat populations by a small amount after employing the contractor for more than a year and despite the contractor putting the full amount of contracted work every month. At about the same time, the local pasture protection board mounted a baiting campaign against foxes on properties adjacent to the national park and found that cats often took these poison baits too. At the end of several more months, the national park management observed, to their delight, that the cat population had suddenly dropped to much lower levels. Which variable in the model would you change to model this scenario?
- (e) Using a bifurcation diagram or otherwise, explain what has happened, mathematically.
- (f) What will happen to the cat population in the national park if the fox baiting program is reduced on the neighbouring properties, assuming that the national park continues to pay a contractor to exterminate feral cats in the park?

(iii) Consider the following nonlinear differential equation;

$$\frac{dx}{dt} = 1 + bx - bx^2 - x^3$$

where b can take on any real value.

- (a) Find all steady states of this equation and note any restrictions on the value of b that are required for the steady states to exist. (*Hint: $x = 1$ is always a root of $1 + (b - 1)x - (b - 1)x^2 - x^3 = 0$.)*)
- (b) Use linear stability analysis to find the stability of all steady states.
- (c) Draw the phase portrait diagrams for $b < -3$, $-3 < b < 1$ and $b > 1$. Hence sketch the bifurcation diagram using b as the bifurcation parameter.