THE UNIVERSITY OF SYDNEY
Differential Equations & Biomathematics

Semester 1 | **Tutorial Week 1** | 2011

COURSE INFORMATION

Current unit information will be made available on the unit web page

`www.maths.usyd.edu.au/u/UG/SM/MATH3063`

Purpose of course: To introduce the qualitative theory of systems of nonlinear ordinary differential equations and to apply this theory to various problems in mathematical biology.

Lecturer: Associate Professor Mary Myerscough, Carslaw room 626,
email: `mary.myerscough@sydney.edu.au`

Tutor: Theo Vo

Tutorials start in Week 2. Rolls will be kept.

Tutorial sheets and solutions will be posted on the unit web page. Please bring your copy to tutorials as no sheets will be handed out. You are expected to prepare for tutorials by revising the lectures and doing the tutorial work in advance.

Consultations will be held on Thursday from 1.05pm to 2pm in Carslaw room 626.

Assessment The final mark will be made up of the following components.

- **:** Examination (*75%*)
- **:** Assignments, one due in Week 5 (*5%*) and one due in Week 11 (*10%*).
- **:** Class test (*10%*) to be held in a lecture in Week 7.

Solutions to selected tutorial exercises and to assignments will be posted on the web as they become available.

Additional information including information about special considerations etc is available in the Senior Mathematics Handbook, which can be downloaded from the Senior Mathematics page `http://www.maths.usyd.edu.au/u/UG/SM/`

Ordinary Differential Equations and Biomathematics Course

: **Week 1** First order equations: Mathematical models–exponential and logistic growth; definition of linear and nonliner, autonomous and non-autonomous. Phase portraits, equilibria, stability (using phase portraits), linear stability, linearisation for single first order equations.

: **Week 2** Bifurcation in single, first order equations, including the expential and logistic models. Harvesting, both constant rate and constant effort. Population catastrophes. Bifurcation diagrams. Hysteresis.

: **Week 3** Introduction to predator-prey models, specifically the Lotka-Volterra model. Nonlinear models cannot be solved explicitly, hence the need for new mathematical tools. Introduction of the phase plane; nullclines, flows, sketching solutions. Phase plane of the Lotka-Volterra equations motivates the need for more information.

: **Week 4** Nonlinear systems and linearisation. Solving linear systems. Classification of the behaviour of linear systems (nodes, focuses, saddles, centres, etc). Phase planes of linear systems. The Jacobian matrix and classifying behaviour using its trace and determinant.

: **Week 5** Phase portraits and linear stability analysis of a variety of nonlinear systems (lots of examples). Existence and uniqueness of solutions. Different types of stability.

: **Week 6** Lotka-Volterra equations using linear analysis and phase planes. Harvesting the Lotka-Volterra equations. Structural instability. Other predator-prey systems. Models for ecological competition, mutualism.

: **Week7** Models for the spread of disease. Definition of an epidemic. Basic SIR model. Critical population sizes. Vaccination effects. What happens as $t \to \infty$. SIS and SIRS models, crisscross infections and STDs.

: **Week 8** Lyapunov stability. Finding and using Lyapunov functions. Sketch of Lyapunov theorems.

: **Week 9** First integrals, Hamiltonian systems and gradient systems. Definition of first integral. The Lotka-Volterra equations as an example of a Hamiltonian system. Conservative systems. nonlinear pendulum, Duffing equation, the Van der Pol oscillator.

: **Week 10** Limit cycles: definition, stability analysis, phase protraits. Biological examples (mainly computational).

: **Week 11** Bifurcation in systems of two first order ODEs. Statement of the Hopf bifurcation theorem. Creation of limit cycles. The Brusselator model. Predator-prey and epidemiological models with Hopf bifurcations.

: **Week 12** Fitzhugh-Nagumo equations, relaxation oscillations and excitable media