Solns Tutorial Week 4. MATH 3063

1 (i)

Nullclines $\frac{dx}{dt}=0$ so $xy=0$ so $x=0$ or $y=0$

$\frac{dy}{dt}=0$ so $(1+x)(1-y)=0$ so $x=-1$, $y=1$.

Flows $\frac{dx}{dt}>0$ if $-xy>0$ $\rightarrow$.

so $x>0$ $y<0$ or $y>0$, $x<0$.

$\frac{dx}{dt}<0$ if $xy<0$

so $x>0$, $y>0$ or $x<0$ and $y<0$

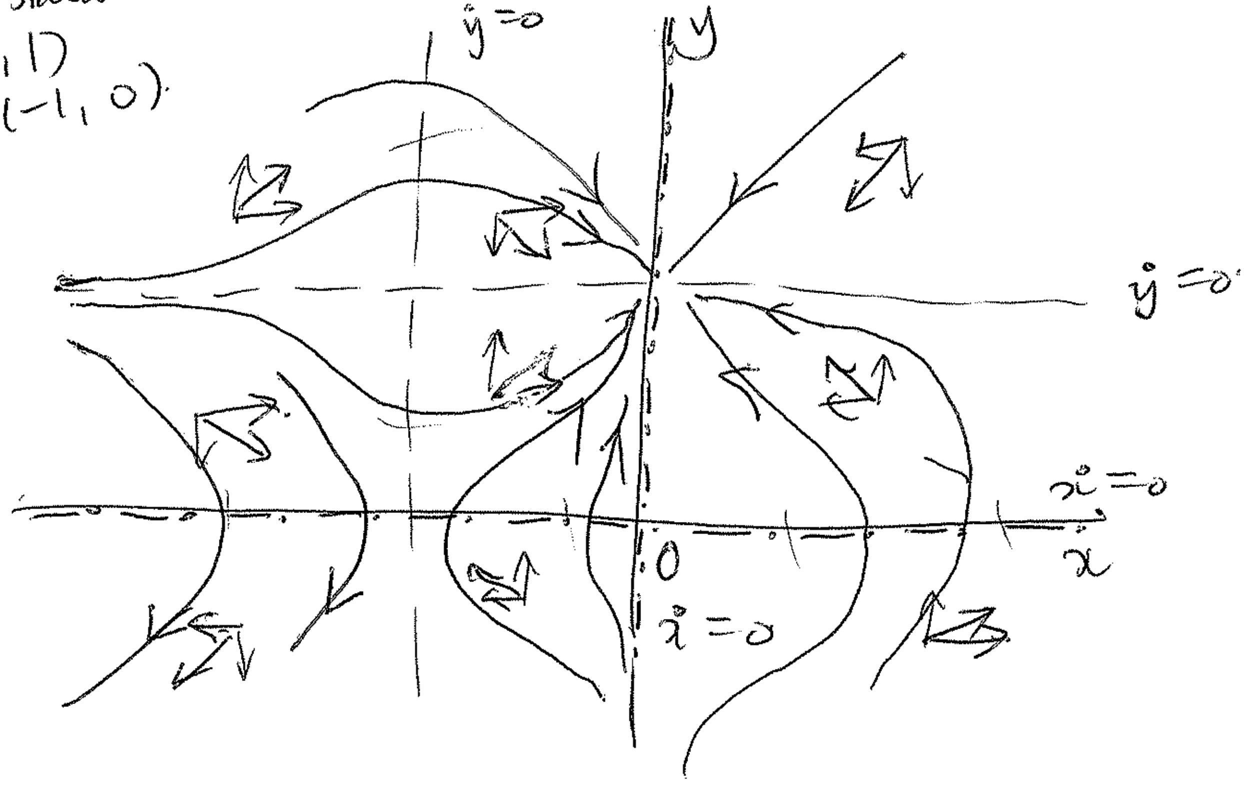
$\frac{dy}{dt}>0$ if $(1+x)(1-y)>0$ so

$x>-1$ and $y<1$. or $x<-1$ and $y>1$.

$\frac{dy}{dt}<0$ if $(1+x)(1-y)<0$ so.

$x>-1$ and $y>1$ or $x<-1$ and $y<1$.

Steady states at $(0,1)$ and $(-1,0)$.



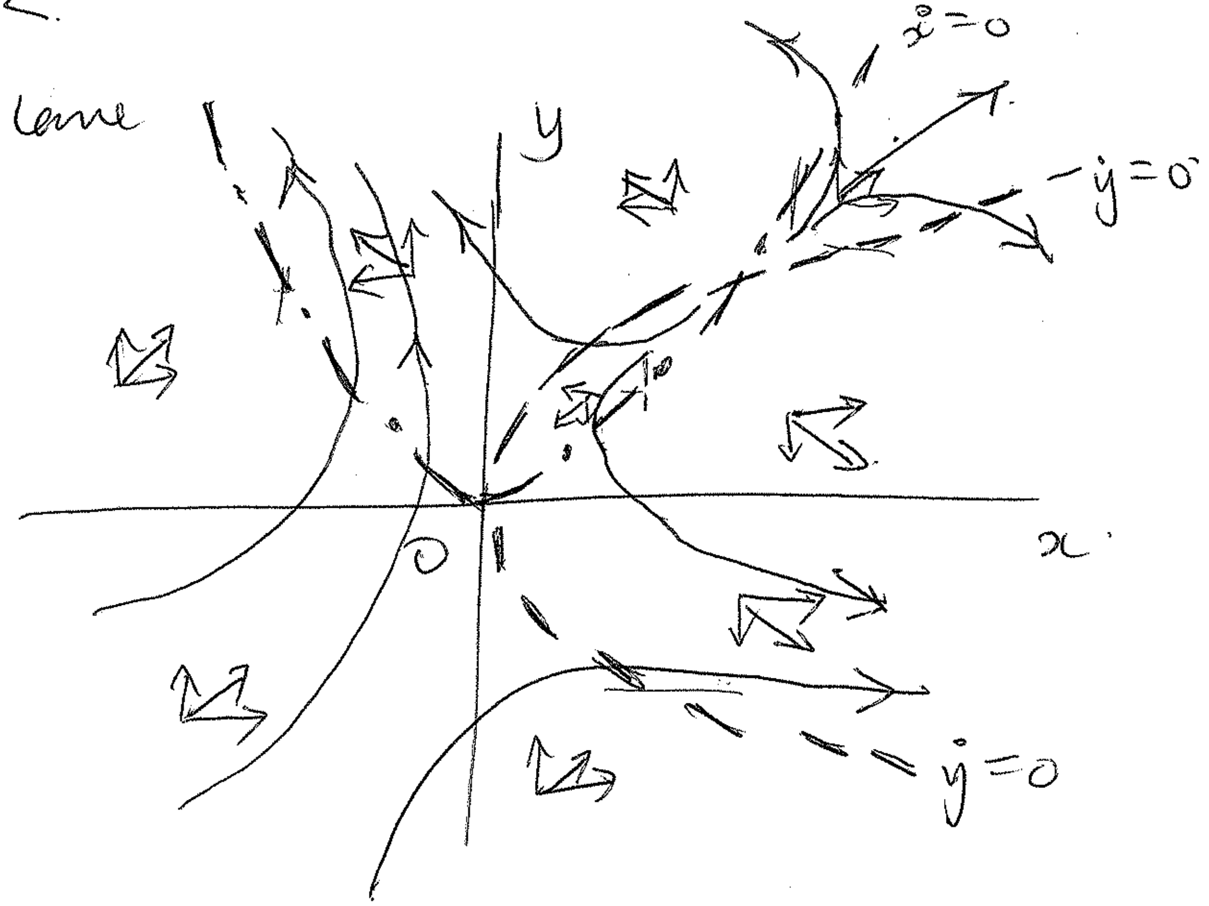
(ii) Nullclines $\frac{dx}{dt} = 0$ so $x^2 - y = 0$ so $y = x^2$.

$\frac{dy}{dt} = 0$ when $y^2 - x = 0$ or $x = y^2$.

Flows $\frac{dx}{dt} \gtrless 0$ when $x^2 - y \gtrless 0$ so $y \lessgtr x^2$

$\frac{dy}{dt} \gtrless 0$ when $y^2 - x \gtrless 0$ so $x \lessgtr y^2$.

Phase plane



Steady states are $(0,0)$ and $(1,1)$

(iii) Nullclines $\frac{dx}{dt} = 0$ $y^2 - x^2 = 0$ so $x = y$,
or $x = -y$.

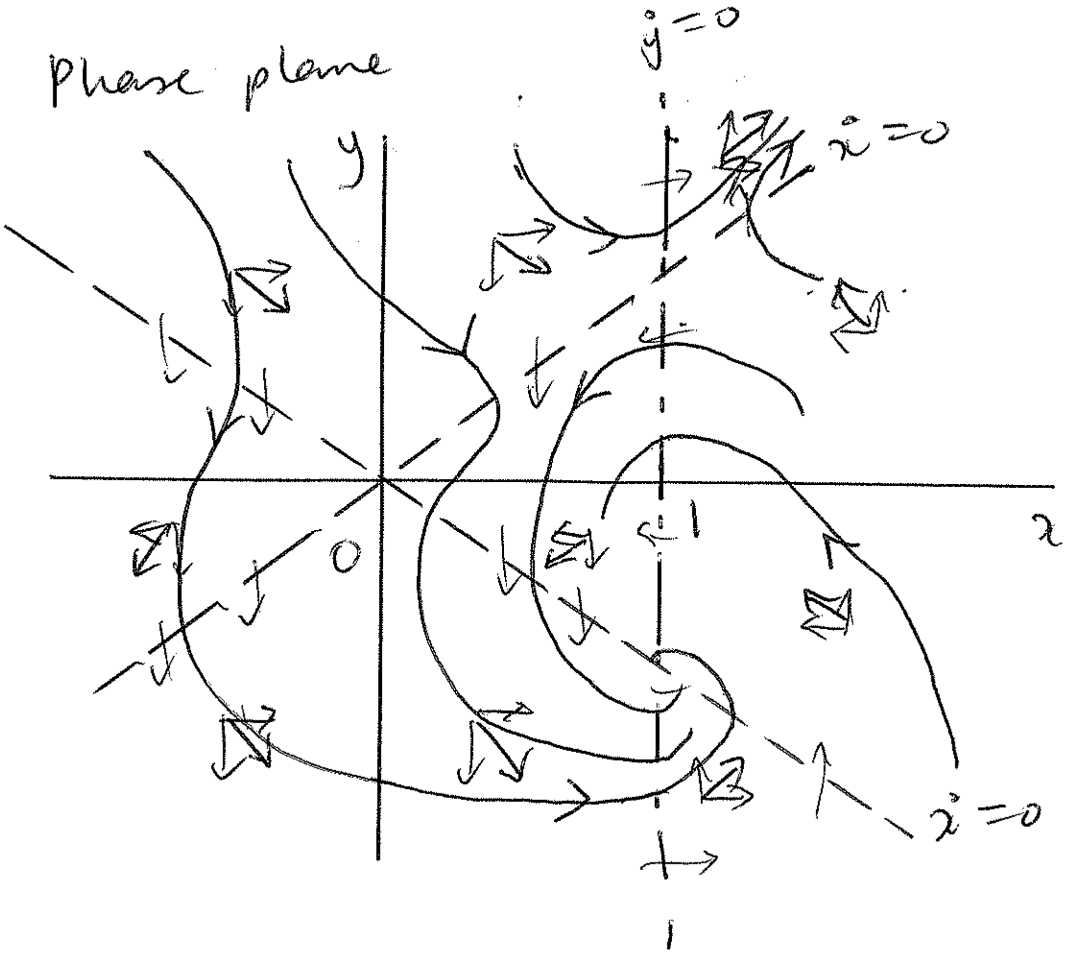
$\frac{dy}{dt} = 0$ so $x - 1 = 0$ $x = 1$.

<u>Flows</u> $\frac{dx}{dt} > 0$ when $y^2 > x^2$.

$\frac{dx}{dt} < 0$ when $y^2 < x^2$.

$\frac{dy}{dt} \gtrless 0$ when $x \gtrless 1$

Phase plane



Steady states at $(1,1)$ and $(1,-1)$.

(iv) Nullclines $\frac{dx}{dt}=0$, $x(y^2-y)=0$

so $x=0$ $y=0$ $y=1$.

$\frac{dy}{dt}=0$ when $x-y=0$ so $x=y$.

Flows $\frac{dx}{dt}>0$ when $x(y^2-y)>0$

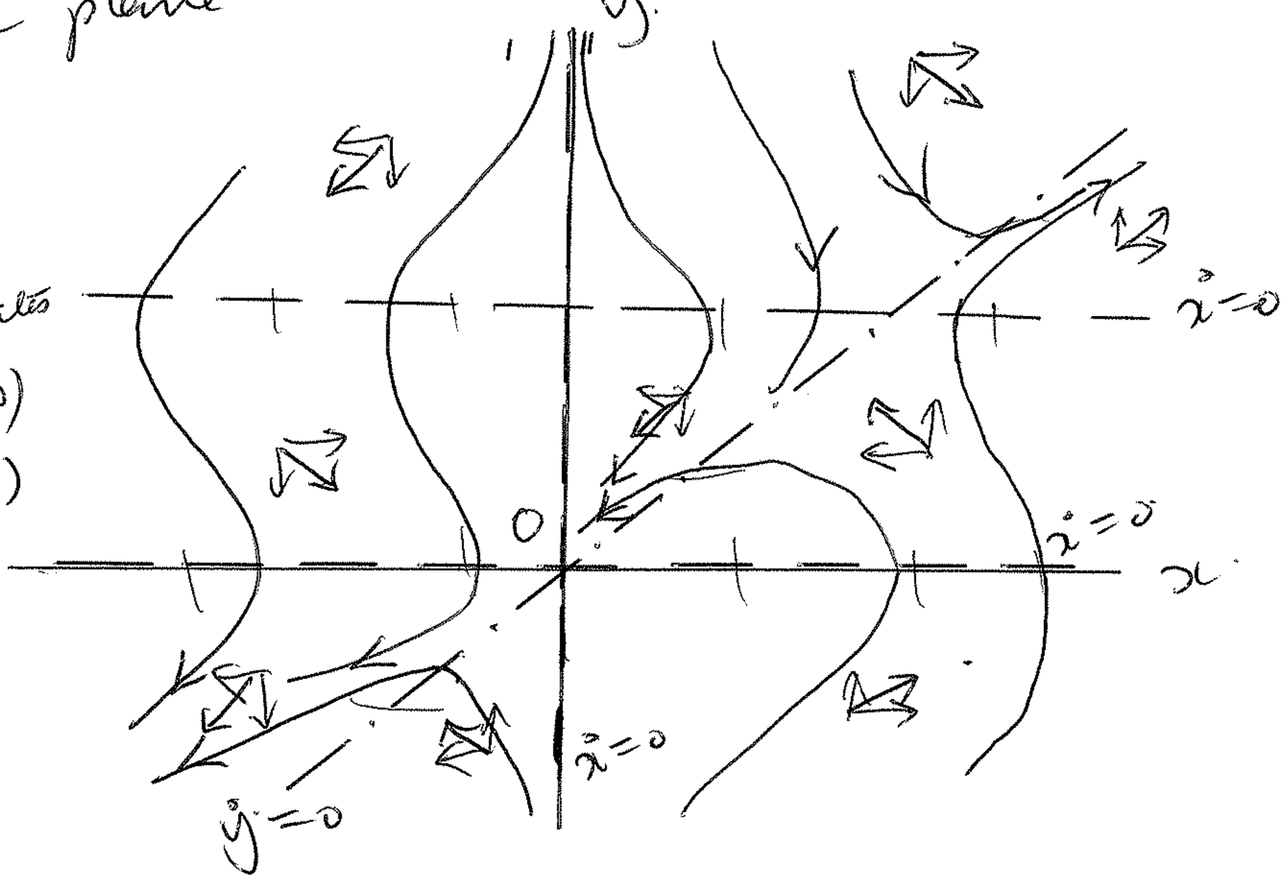
so $x>0$, $y^2-y>0$ i.e. $y<0$ or $y>1$ <u>or</u> $x<0$ and $0<y<1$.

$\frac{dx}{dt}>0$ when $x<0$ and $y<0$ or $y>1$.
and $x>0$ and $0<y<1$.

$\frac{dy}{dt} \gtrless 0$ when $x \gtrless y$

phase plane.

Steady states at $(0,0)$ and $(1,1)$



(v). Nullclines $\frac{dx}{dt} = 0$. $x\left(\frac{-y}{1+x} + 1\right) = 0$

so $x = 0$ or $y = 1 + x$.

$\frac{dy}{dt} = 0$ $y\left(\frac{x}{1+x} - 1\right) = 0$.

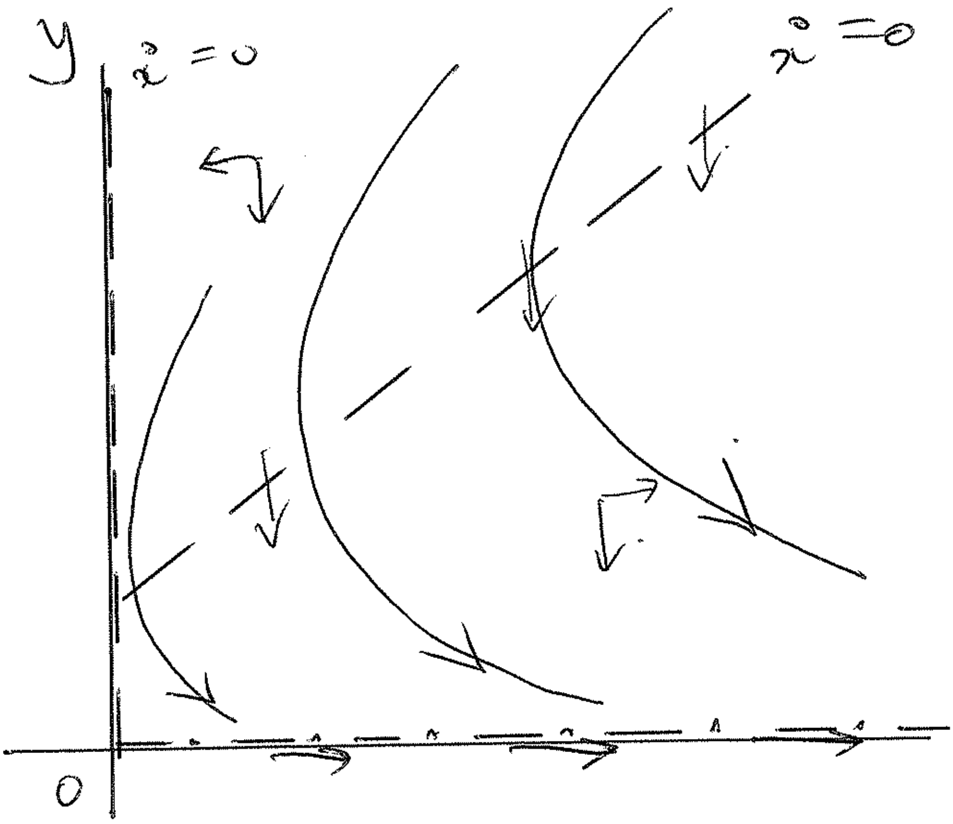
so $y = 0$ or $x = 1 + x$ — not a soln.

Flows $\frac{dx}{dt} > 0$ if $y < 1 + x$. (Note $x \geq 0$!)

$\frac{dx}{dt} < 0$ if $y > 1 + x$.

$\frac{dy}{dt} > 0$ if $\frac{x}{1+x} - 1 > 0$ BUT $\frac{x}{1+x} < 1$ for $x > 0$.

so $\frac{dy}{dt} < 0$ for all $x, y > 0$.



(vi) Nullclines: $\frac{dx}{dt} = 0$ $\quad xy(1-x)+c=0$

so $\quad xy(1-x) = -c \quad \boxed{\text{If } c \neq 0}$

so if $x \neq 0$ or 1 $\quad y = \frac{-c}{x(1-x)}$

$\frac{dy}{dt} = 0$: $\quad y(1 - y/x) = 0$ so $y = 0$.

or $y = x$ and $x \neq 0$.

Flows: $\frac{dx}{dt} > 0$ if $xy(1-x)+c > 0$

So if $x < 0$ $\quad x(1-x) < 0$ so $y < \frac{-c}{x(1-x)}$.

if $0 < x < 1$ $\quad x(1-x) > 0$ so $y > \frac{-c}{x(1-x)}$.

$x > 1$ $\quad x(1-x) < 0$ so $y < \frac{-c}{x(1-x)}$.

$\frac{dx}{dt} < 0$ if $xy(1-x)+c > 0$

If $x < 0$ $\quad y > \frac{-c}{x(1-x)}$

$0 < x < 1$ $\quad y < \frac{-c}{x(1-x)}$.

$x > 1$. $\quad y > \frac{-c}{x(1-x)}$.

$\frac{dy}{dt} > 0$ if $y > 0$ and $1 - y/x > 0$. $1 > y/x$

so $x > y$ for $x > 0$

$x < y$ for $x < 0$.

or if $y < 0$ and $x < y$ for $x > 0$ (impossible

$x > y$ for $x < 0$

$\frac{dy}{dt} < 0$ if $y > 0$ and $1 - y/x < 0$

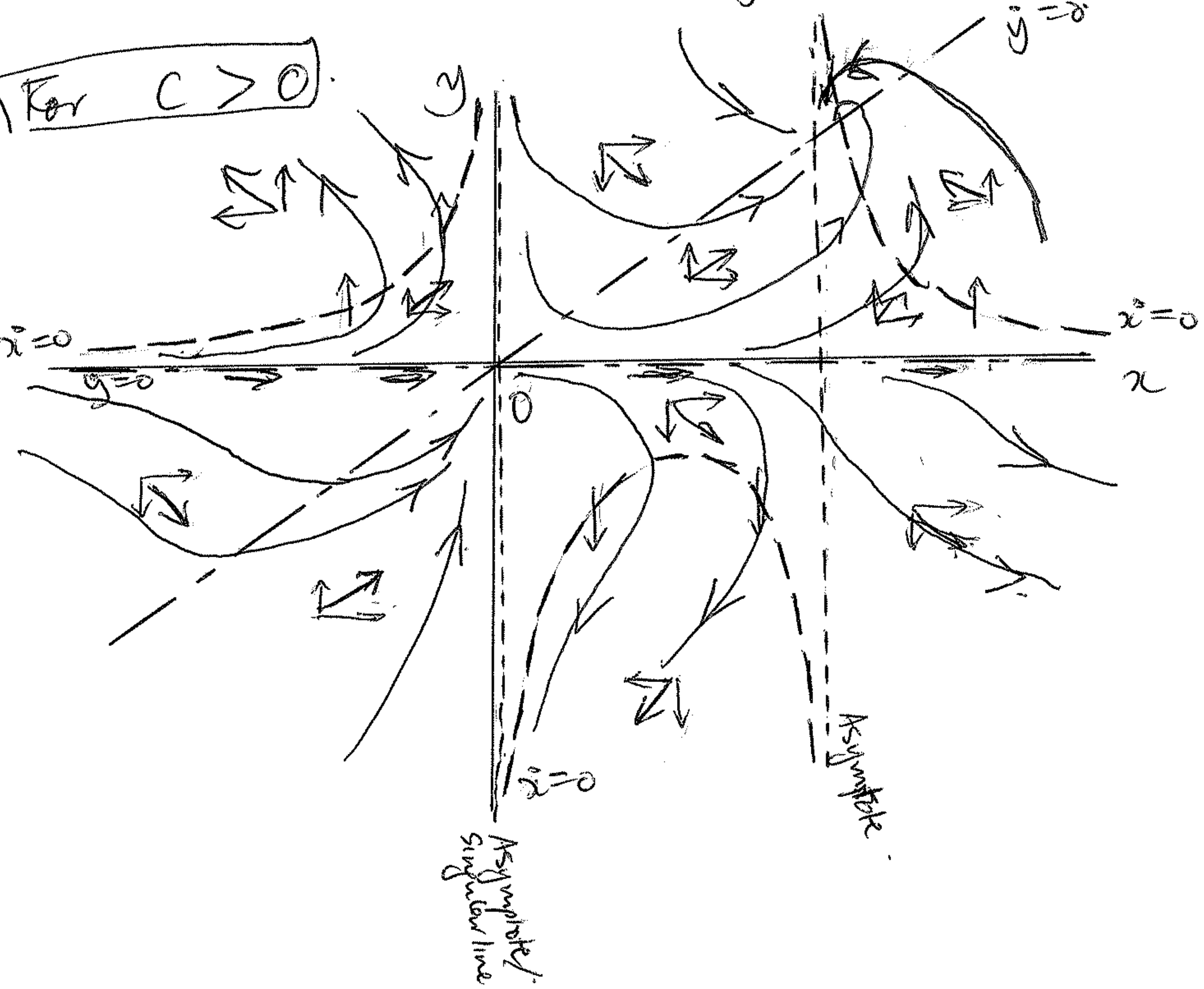
so $x < y$ for $x > 0$

$x > y$ for $x < 0$

or if $y < 0$ and $x > y$ for $x > 0$

$x < y$ for $x < 0$

For $c > 0$



Note: on $x = 0$ dy/dt is not defined. The asymptote $x = 1$, however does not represent any discontinuity in the phase flow (unlike $x = 0$.)

Steady state at $x^* = y^* = \frac{-c}{x^*(1-x^*)}$.

$(0,0)$ is singular - not a steady state

$\boxed{\text{For } c = 0}$ $\frac{dx}{dt} = 0$ when $xy(1-x) = 0$ so

$x = 0$, $y = 0$ $x = 1$.

Flow $\frac{dx}{dt} > 0$ when $x > 0, y > 0$ $x < 1$.

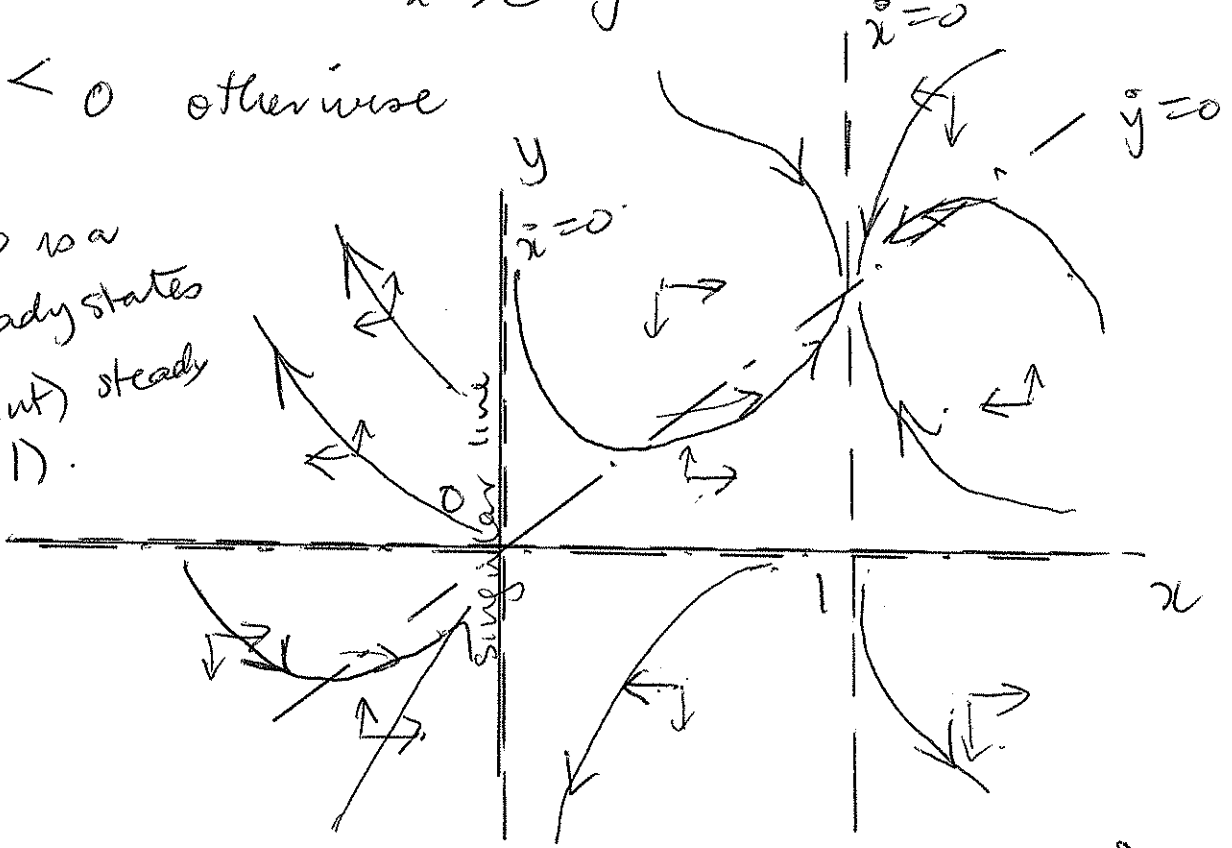
or $x < 0$ $y > 0$

or $x > 0$ $y < 0$.

$\frac{dx}{dt} < 0$ otherwise

Note $y = 0$ is a <u>line</u> of steady states

Another (point) steady state at $(1, 1)$.



For $c < 0$

Two steady states

one for $x > 0, y > 0$

one for $x < 0, y < 0$.