

1. Revise solving separable and first order linear differential equations.

Find explicit solutions of the equations

$$\begin{array}{ll} (i) \quad \dot{x} = x(2t - 1), & (ii) \quad \dot{x} - x = e^t, \\ (iii) \quad \dot{x} + tx = t, & (iv) \quad \dot{x} = x(1 - x). \end{array}$$

2. Determine the equilibrium points, draw the phase line and classify the equilibrium points of each of the growth laws below.

$$\begin{array}{ll} (i) \quad \dot{N} = \alpha N^2 - \beta N, & (\alpha, \beta > 0, \text{ constant}), \\ (ii) \quad \dot{N} = \alpha N + \beta N^2, & (\alpha, \beta > 0, \text{ constant}), \\ (iii) \quad \dot{N} = rN(N/M - 1)(1 - N/K), & \\ & (r > 0, 0 < M \ll K, \text{ and } r, M, K \text{ all constant}), \\ (iv) \quad \dot{N} = rNe^{-\beta N}, & (r, \beta > 0). \end{array}$$

3. Solve the logistic equation growth equation  $\dot{x} = rx(1 - x/K)$ ,  $r, K > 0$  positive constants, using separation of variables and show that

$$x(t) = \frac{Kx(0)}{x(0) + (K - x(0))e^{-rt}},$$

where  $x(0)$  is the initial population. Sketch the solution curves for various choices of  $x(0)$ . Using the explicit solution above, determine analytically the behaviour of solutions for  $x(0) > K$  and  $0 < x(0) < K$ .

4. Consider the area of Australia colonised by cane toads (*Bufo marinus*), tabulated in 5 yearly intervals 1939-1969.

Year	Total Km <sup>2</sup> Colonised
1939	32,800
1944	55,800
1949	73,600
1954	138,000
1959	202,000
1964	257,000
1969	301,000

- (i) Let  $A$  be the area in thousands of square kilometres that is occupied by cane toads as a function of  $t$  the time in decades since 1939. Assume that the rate of growth of this area is proportional to the area that is already occupied by cane toads. Formulate a differential equation model for the area colonised. Solve your equation and use the data points of 1939 and 1969 to find the model parameters. Predict the land area occupied in 2009 and 2019. Do you believe these predictions? Why or why not? (*Remark* The area of Australia is 7,619,000 km<sup>2</sup>.)
- (ii) Assume the growth of area colonised follows a logistic model with carrying capacity the area of Australia. Estimate the parameter  $r$  using the data for 1969. Use the model to predict when half the area of Australia will be occupied by cane toads.