

1. Consider the predator(y)-prey(x) model

$$\dot{x} = x(a - \ell x - by), \quad \dot{y} = y(-c + dx)$$

- (i) Calculate all possible equilibrium solutions and compare with the ones which occur when $\ell = 0$. Briefly explain the differences between the two cases $\ell = 0$, $\ell \neq 0$.
- (ii) Draw isoclines of the system in the phase plane and sketch roughly the solution trajectories.
- (iii) Do a linearised stability analysis of the non-zero equilibrium populations.
2. Larkin (*Journal of the Fisheries Research Board of Canada* **23**, 349 (1966)) proposed the following predator(y)-prey(x) model:

$$\dot{x} = rx(1 - x/K) - \alpha xy, \quad \dot{y} = sy(1 - y/L) + \beta xy; \quad r/\alpha > L.$$

Sketch the isoclines and solution trajectories in the first quadrant of the phase plane. Do a linearised stability analysis of the non-zero equilibrium state.

3. Two types of yeast x and y grow in competition in a medium in the absence of oxygen. Yeasts produce alcohol such that the resulting alcoholic content is approximately proportional to the total number of yeast ($x + y$). However, the growth rates of the yeasts are increasingly retarded as the alcohol content increases. This is modelled by

$$\begin{aligned} \dot{x} &= x(a - b(x + y)) \\ \dot{y} &= y(c - d(x + y)). \end{aligned}$$

Sketch the phase trajectories for the case $c/d > a/b$ and hence deduce which yeast has the highest tolerance of alcohol.

4. Ref: C.W. Clark, *Mathematical Bioeconomics*, pp 205–207.

This exercise illustrates some of the risks of resource exploitation. One could inadvertently drive a species to extinction because of its (perhaps unknown) interaction with some other species.

Consider two species x and y growing in competition so that in the natural state

$$\begin{aligned} \dot{x} &= rx(1 - x/K) - \alpha xy \\ \dot{y} &= sy(1 - y/L) - \beta xy. \end{aligned}$$

Now suppose x is harvested at constant effort E .

Using a phase plane analysis, investigate the equilibria as E increases if $r/\alpha > L$ and $s/\beta > K$.