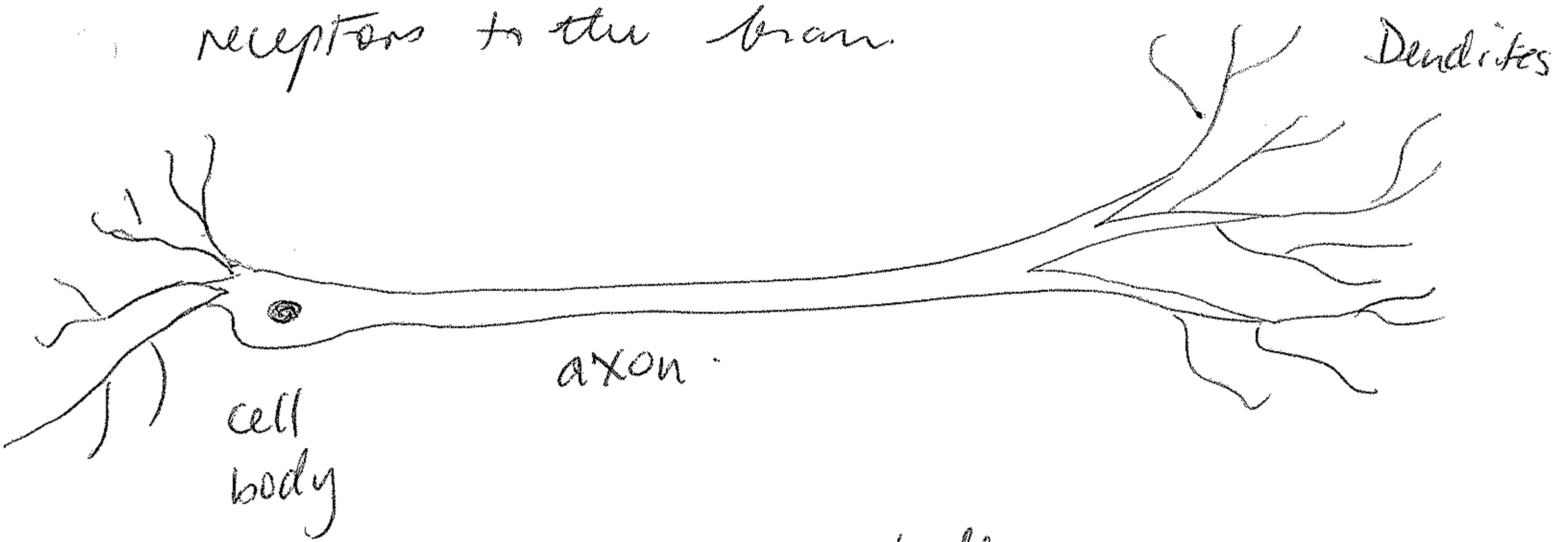
8 Mathematical Models for Nerve Conduction:

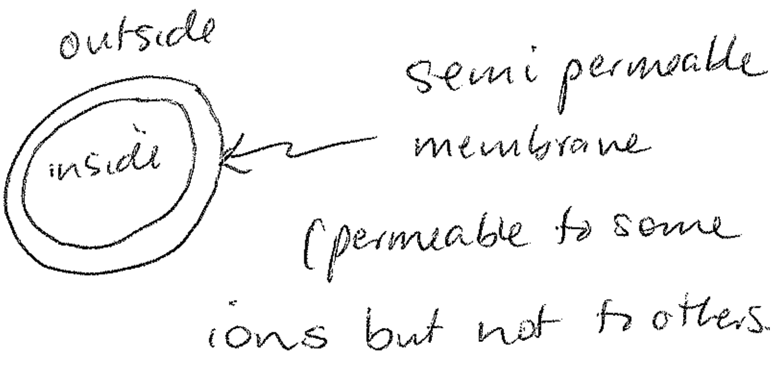
8.1 Basic neural physiology

An example of mathematics applied in physiology:

Electrical impulses travel along nerve axons connecting muscles and sensory receptors to the brain.



Cross-section of axon

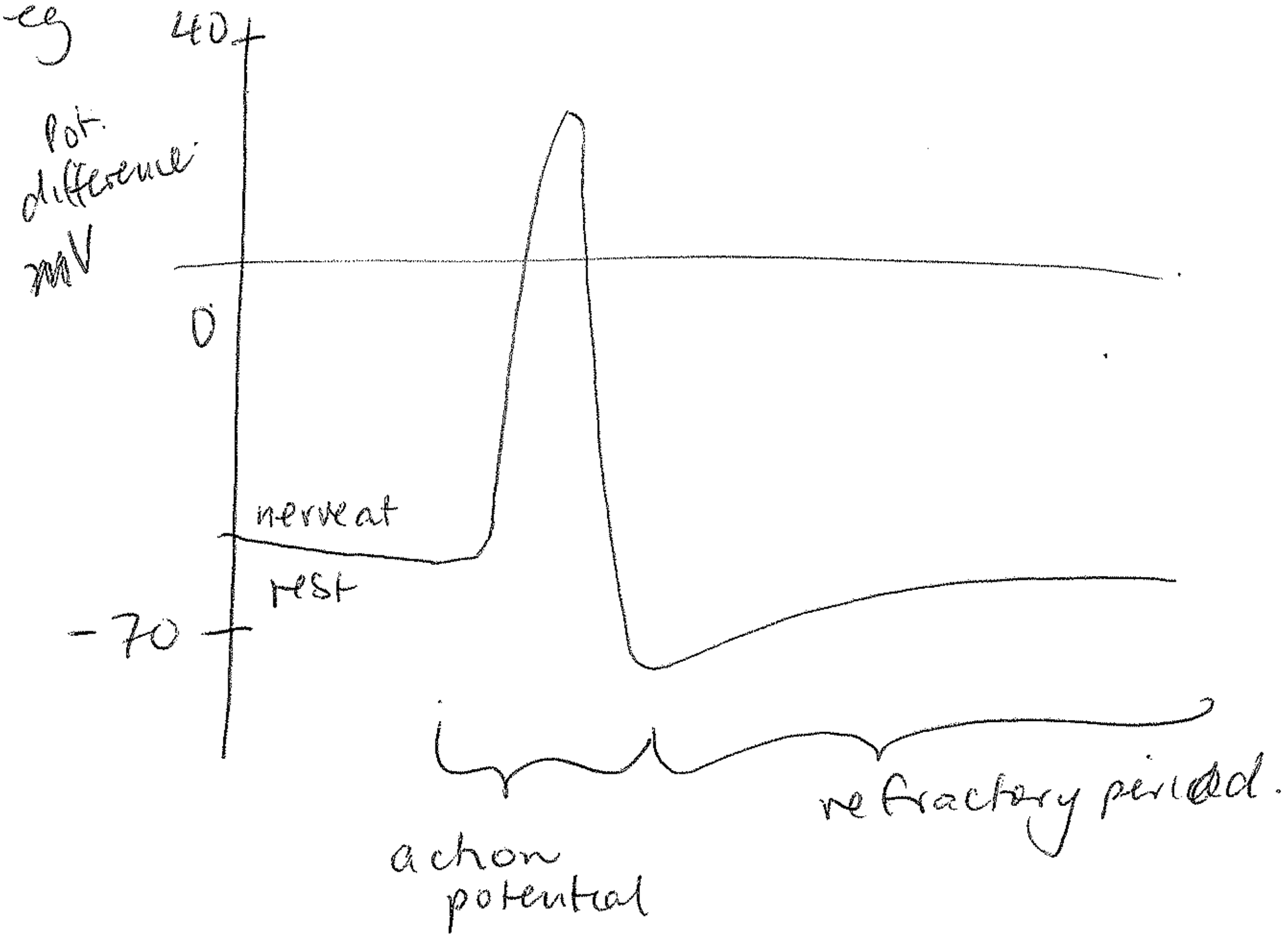


Ionic channels in the membrane allow ions to flow in and out.
Each channel is particular to a single type of ion (eg Na^+ or K^+)

Whether or not a channel is open depends on the <u>electrical potential</u> across the membrane.

When a nerve "fires" the potential difference across the membrane changes to give an <u>action potential</u>.

eg



What happens during an action potential?

Look at Na^+ (sodium) and K^+ (potassium) ions in the neurone of the giant squid.

Note membrane potential = electrical pot. inside − electrical pot. outside

At rest.

Low [Na^+]
High [K^+]
−70 mV.

Membrane closed to Na^+

Open to K^+.

During action potential – at the start.

Increasing [Na^+]
High [K^+]
$-70 \to 40$ mV

Na^+ Sodium flows in rapidly

Membrane opens to Na^+.

During end of action potential and refractory period.

Med [Na^+]
~~Decreasing~~ [K^+]
40 mV

K^+

Potassium flows out

Permeability to Na^+ drops as membrane potential drops.

Finally cell uses cellular "pumps" to re-equilibrate ionic balance.

<u>Fast events</u> – rise in membrane potential, opening of Na^+ channels.

<u>Slow events</u> – deactivation of Na^+ channels, activation of K^+ channels

8.2 FitzHugh - Nagumo model for

Neural conduction:

We consider

Fast activating variable v (Membrane potential Sodium channels)

Slow, recovery variable w (Opening of Potassium channels, Shutting of Na^+ channels)

Two equations.

"Membrane potential" $\frac{dv}{dt} = -v(v-a)(v-1) - w$

$0 < a < 1.$

"Recovery variable" $\frac{dw}{dt} = \varepsilon(v - w) \qquad \varepsilon << 1.$

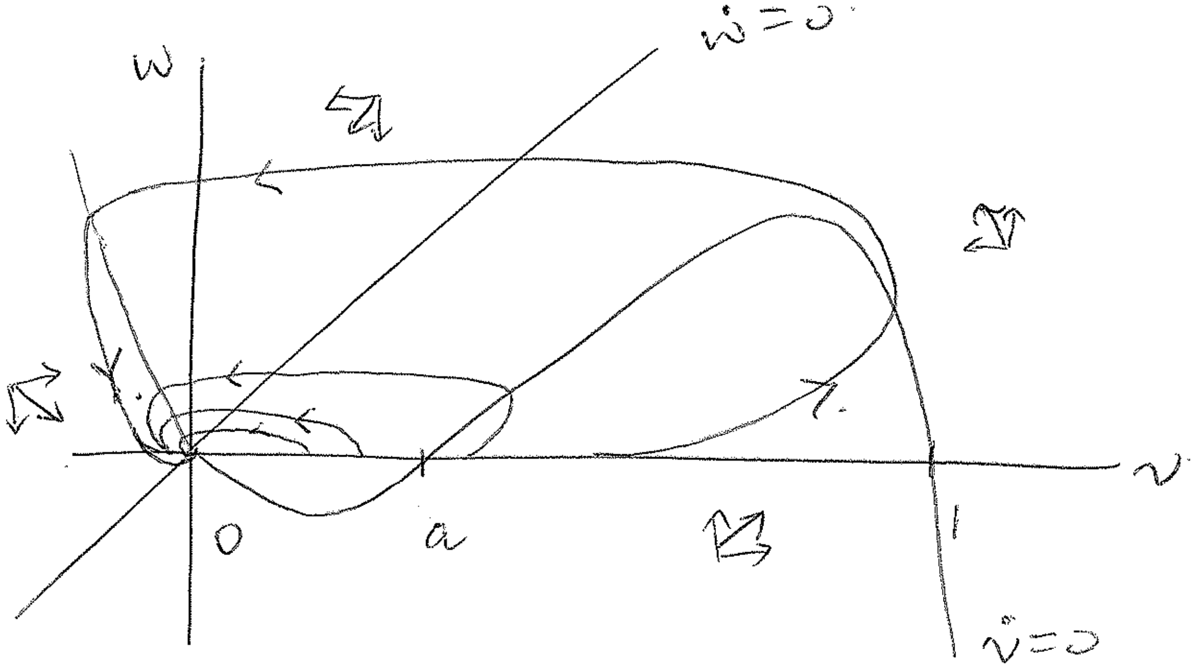
Phase plane Nullclines $\dot{v} = 0 \quad w = -v(v-a)(v-1).$

$\dot{w} = 0 \quad w = v.$

Flows $\dot{v} \gtrless 0$ if $w \lessgtr -v(v-a)(v-1).$

$\dot{w} \gtrless 0$ if $w \lessgtr v.$

Note both v and w can be negative.



One steady state $(0,0)$.

Now since $\frac{dw}{dt} = \varepsilon(v-w) \propto \varepsilon \ll 1$, then the flow in w direction is very slow. of order $O(\varepsilon)$.

Near $\dot{v}=0$ nullcline $\frac{dv}{dt} = O(\epsilon)$ (very small) but otherwise $\frac{dv}{dt}$ is $O(1)$ (normal speed).

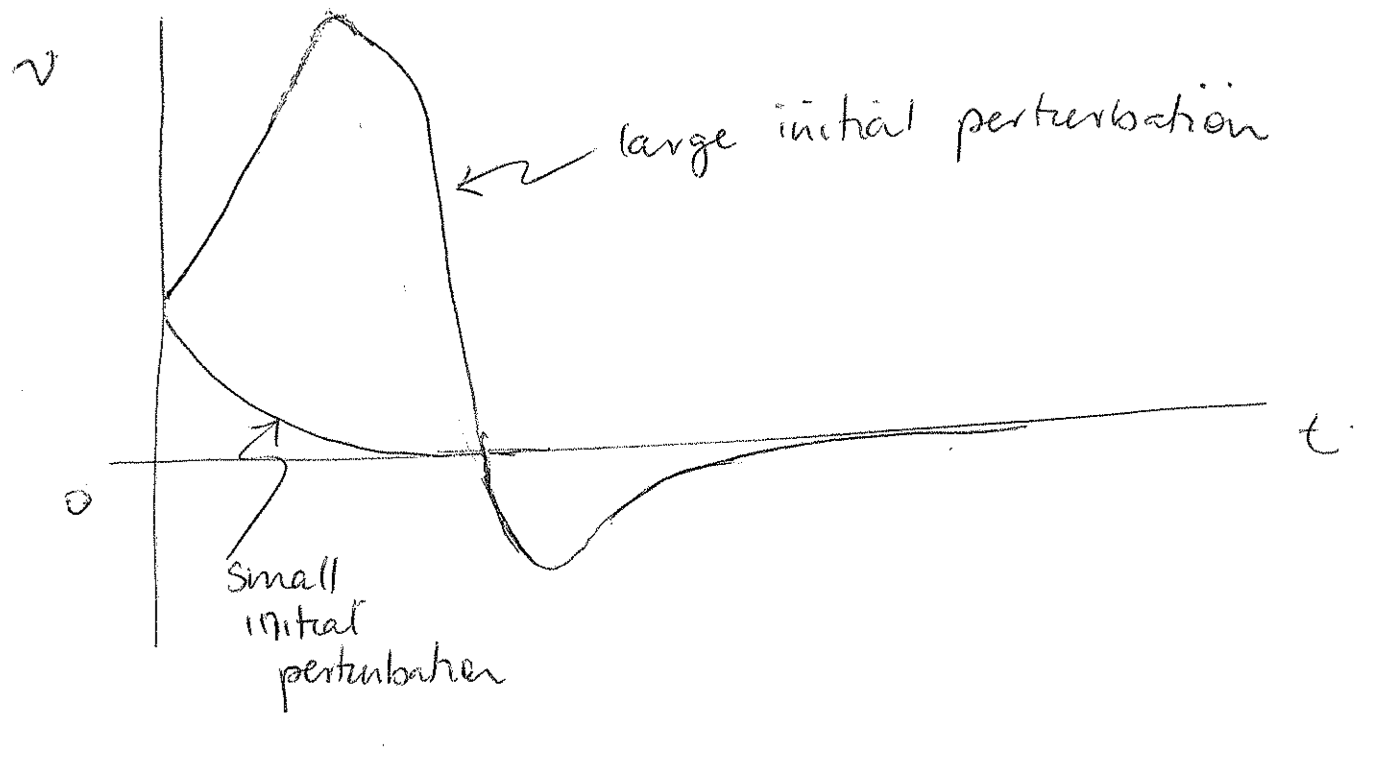
So away from $\dot{v}=0$ nullcline, flow is much faster horizontally than vertically So trajectories are nearly horizontal

Start with initial condition $w = 0, v = v_0$.

If v_0 is small system quickly returns to steady state.

If v_0 is larger, system goes on a long excursion before returning to $(0,0)$.

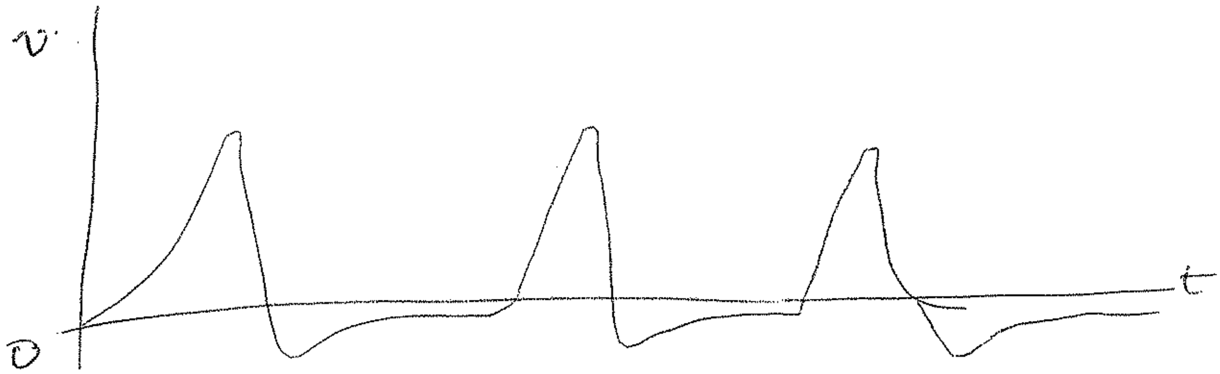
This type of behaviour is called <u>threshold behavior</u>



8.3. Adding an applied current.

Effects + a current is applied to lower membrane potential. When the current is switched off - nerve fires! (Anode break excitation

2. If a nerve is stimulated with an applied current I, then it fires repeatedly
which raises membrane potential



Change the FHN eqns to include I.

$$\frac{dv}{dt} = -v(v-a)(v-1) - w + I.$$

$$\frac{dw}{dt} = \varepsilon(v-w).$$

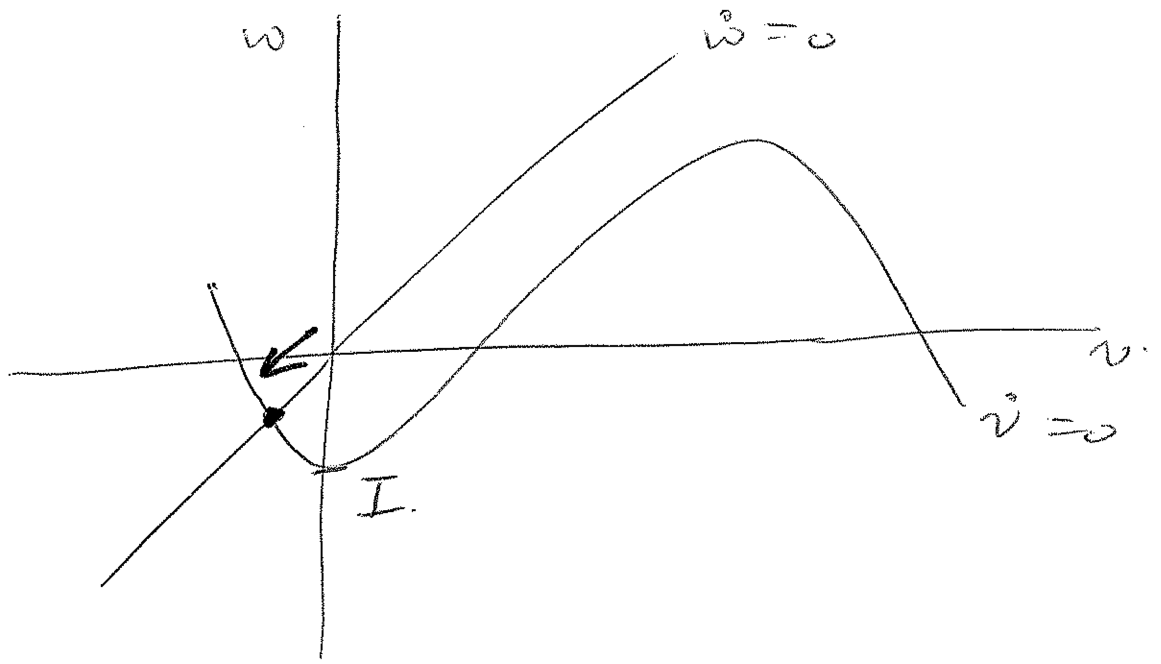
I can be +ve or -ve.

New $\dot{v}=0$ nullcline is

$$w = -v(v-a)(v-1) + I.$$

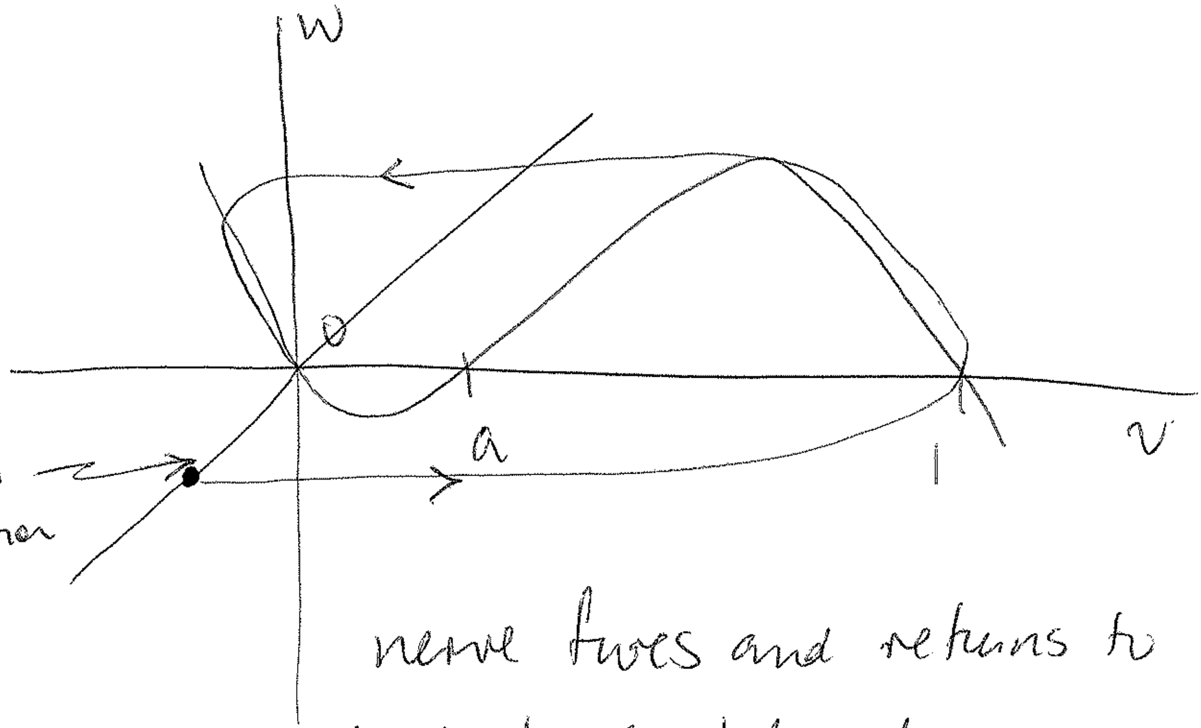
1. Anode break excitation,

Impose $I < 0$ on system



→ steady state moves from $v=0$ to $v<0$.

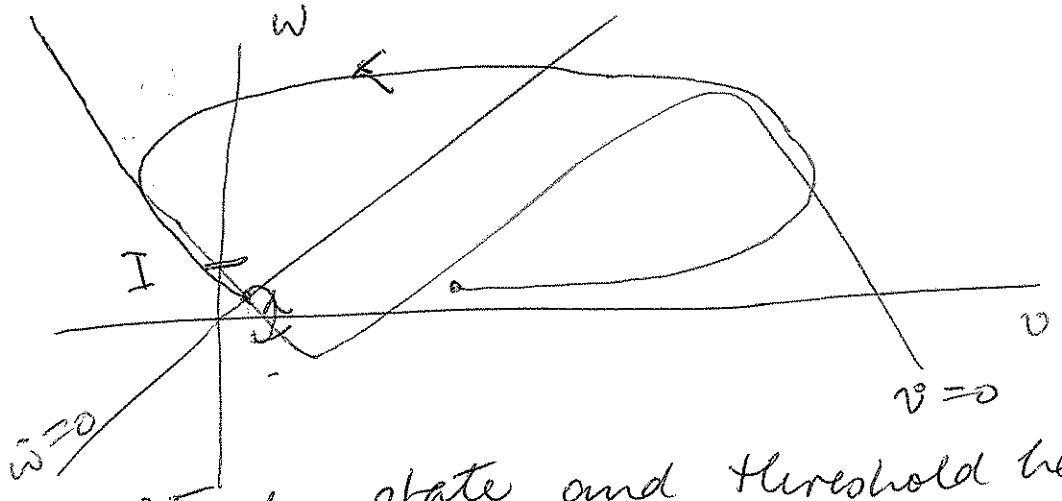
When system has settled to steady state set $I=0$



nerve fires and returns to new steady state at $v=0$.

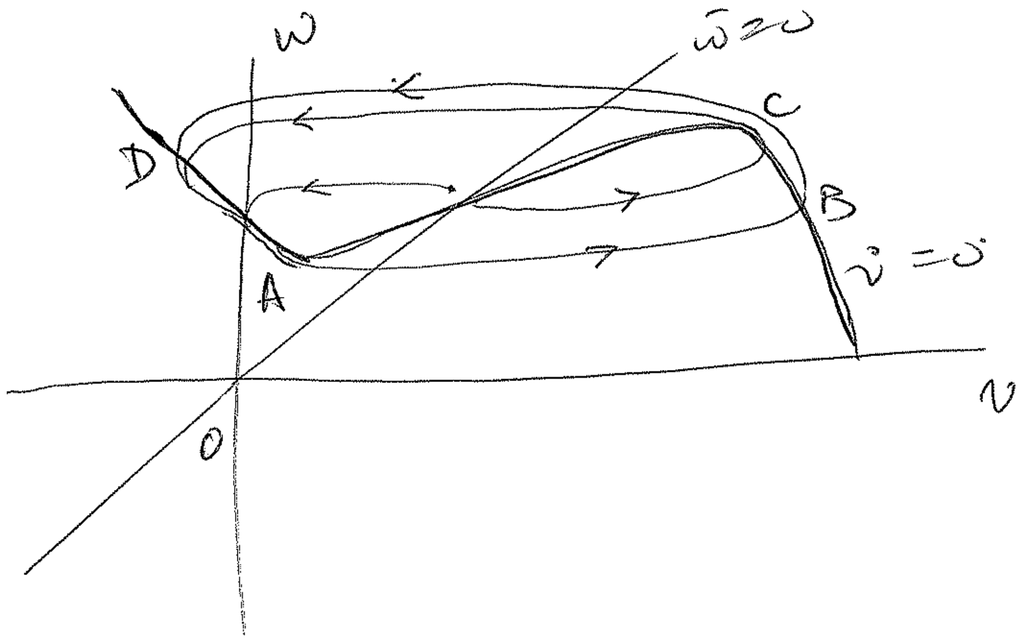
2. Oscillations.

For small I., I > 0

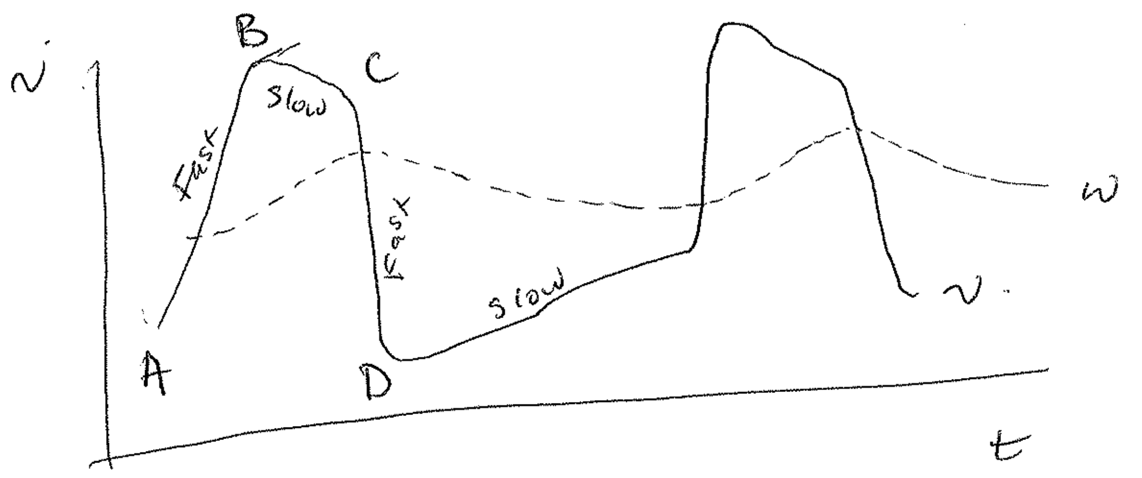


Steady state and threshold behaviour as before.

Increasing I gives steady state on middle branch of cubic



Now steady state is unstable and we have a limit cycle that flows along the outer branches of cubic

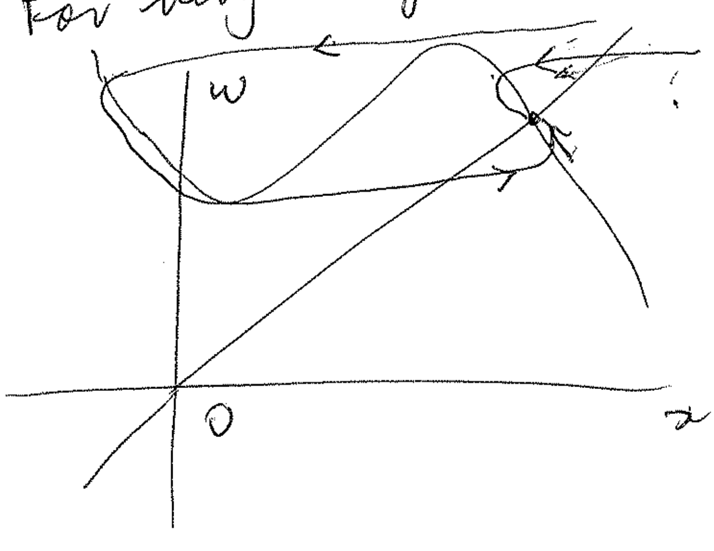


This oscillation is known as a <u>relaxation oscillation</u>.

Relaxation oscillations and threshold behaviours are typical of <u>excitable media</u>

Excitable media include nerves, electrically active muscle (eg heart muscle), some other cells (eg pancreatic β-cells) the Belusov - Zhabotinsky reaction mixture, ~~etc~~. aggregating slime moulds, crowds doing Mexican waves......

For very large I



Stable steady state exists again.