Time allowed: two hours

No notes, books or calculators are allowed.

Instructions to candidates:

This examination paper comprises six questions, worth 20, 20, 20, 20, 30, 30 marks respectively.

The total number of marks available is 140.

Full marks may be awarded for achieving 120 or more marks.

All questions may be attempted.

Questions 1 and 2 relate to the Propositional Calculus.

Questions 3 and 4 relate to the Predicate Calculus.

Question 5 relates to Turing machines.

Question 6 relates to sets and mappings.

For reference, on the last page of the examination paper, there is a list of the ten rules of deduction for the Propositional Calculus, followed by the further four rules of deduction for the Predicate Calculus.
The Ten Rules of Deduction for the Propositional Calculus:

1. **Rule of Assumptions (A):** Any wff may be written down as an assumption, depending only on itself.

2. **Modus Ponens (MP):** Given $V$ and $V \Rightarrow W$, we may deduce $W$, depending on the pooled assumptions for $V$ and $V \Rightarrow W$.

3. **Modus Tollens (MT):** Given $\sim W$ and $V \Rightarrow W$, we may deduce $\sim V$, depending on the pooled assumptions for $\sim W$ and $V \Rightarrow W$.

4. **Double Negation (DN):** Given $\sim\sim W$, we may deduce $W$, and vice-versa, in each case depending on the same underlying assumptions.

5. **Conditional Proof (CP):** Given $V$, introduced earlier by Rule of Assumptions, and given $W$, relying on $V$, we may deduce $V \Rightarrow W$, discharging $V$, but relying on any remaining assumptions used to deduce $W$ from $V$.

6. **\&-Introduction (\&I):** Given $V$ and $W$, we may deduce $V \& W$, relying on the pooled assumptions for $V$ and $W$.

7. **\&-Elimination (\&E):** Given $V \& W$, we may deduce $V$ or deduce $W$, relying on assumptions for $V \& W$.

8. **\lor-Introduction (\lor I):** Given $V$, we may deduce $V \lor W$ or deduce $W \lor V$ for any $W$, relying on the assumptions for $V$.

9. **\lor-Elimination (\lor E):** Given $V \lor W$ and two deductions of $C$, firstly from $V$, introduced by Rule of Assumptions, and secondly from $W$, introduced by Rule of Assumptions, we may deduce $C$ again, but from $V \lor W$, discharging the assumptions $V$ and $W$, but pooling any assumptions for $V \lor W$ and any assumptions used to deduce $C$ from $V$ and $C$ from $W$.

10. **Reductio ad Absurdum (RAA):** Given $V$, introduced earlier by Rule of Assumptions, and given the contradiction $W \& \sim W$, relying on $V$ as an underlying assumption, we may deduce $\sim V$, discharging the assumption $V$, but relying on any remaining assumptions used to deduce $W \& \sim W$ from $V$.

The Four Extra Rules of Deduction for the Predicate Calculus:

11. **\forall-Introduction (\forall I):** Given a wff $W(b)$, where $b$ is a constant symbol that occurs at least once, we may deduce $(\forall x) W(x)$, where $x$ is a new variable that does not appear in $W(b)$ and replaces $b$ uniformly throughout $W(b)$, relying on the assumptions for $W(b)$, provided the symbol $b$ does not appear in any wff in this list of underlying assumptions.

12. **\forall-Elimination (\forall E):** Given a wff $(\forall x) W(x)$, we may deduce $W(b)$, where $b$ is a constant symbol replacing $x$ uniformly throughout $W(x)$, relying on assumptions for $(\forall x) W(x)$.

13. **\exists-Introduction (\exists I):** Given a wff $W(b)$, where $b$ is a constant that occurs at least once, we may deduce $(\exists x) W(x)$, where $W(x)$ results from $W(b)$ by replacing at least one occurrence of $b$ by $x$, relying on assumptions for $W(b)$.

14. **\exists-Elimination (\exists E):** Given a wff $(\exists x) W(x)$ and a deduction of $C$ from $W(b)$, introduced by Rule of Assumptions, where $b$ is a new constant symbol that replaces $x$ uniformly throughout $W(x)$, we may deduce $C$ again, but from $(\exists x) W(x)$, discharging the assumption $W(b)$, but pooling any assumptions for $(\exists x) W(x)$ and any assumptions used to deduce $C$ from $W(b)$, provided $b$ does not appear in $C$ or in any of these underlying assumptions.

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