Logic & Foundations

Lecturer: David Eastham
Lectures: Mon, Wed, Thu 10 am

tutorials (from Week 2): Wed, Thu 9 am

10 mins to fill out profile sheets

Collection of results on board

- indicating general background
- any preknowledge & key ideas

Whilst collecting results, student can complete the following exercise:

**Announcement:** There will be a surprise quiz next week on the Mon, Wed or Thu.

**Corollary:** There is no quiz next week.
Proof. (i) If the quit is an Thurs then you
know after Wed is no surprise
Hence no quit  an Thurs

(ii) If the quit is a Wed then you,
know after Mon is no surprise
Hence no quit  an Wed

By (i) & (ii), quit must be an Mon
is no surprise
Hence no quit next week.

Hence 2+2 = 5. Hence is a quit next week!!

Can a person announcing be believed?

2+2 = 5

I am reliable i.e. changes tell the truth.

Temporal-epistemological logics??
Computes assigned proof that there is no quit next week:

- Google MATCH 3025
- Introductory notes on implication
- Week 2 Exercises (sharing system)
- Assessment into (note: no quit next week
  no quitter at all!!!)
  Q.E.D

Discuss class profile & some key ideas

Modus ponens: rule of inference

\[
\begin{align*}
A & \implies B \\
\therefore B
\end{align*}
\]

- One idea in proofs in Propositional Calculus
- Mathematical implication explained in notes
- Underlying semantics (truth values)

syntax & proofs
Quantifiers: \( \forall, \exists \)

"for all", "for some" or "there exists" leads to Predicate Calculus

Predicates or properties of objects

\( \forall, \exists \) allow quantities to be captured succinctly

Predicate Calculus also has

- Syntax, i.e. the form of deductive proof
- Semantics, meaning, truth values
- Sound: everything deducible is true
- Complete: everything that is true is deducible

Gödel's incompleteness theorem 1929.
Zorn's Lemma equivalence to the Well-Ordering Principle
(see page 165: every set can be well-ordered)

No effective well-ordering of \( \mathbb{R} \) is known!!

(We can easily well-order \( \mathbb{Q} \) however.)

Part 1: Set Theory

Inherently incomplete: see Russell's Paradox

Gödel's Incompleteness Theorem (1931)
Russell's Paradox

Let \( S = \{ x \mid x \notin x \} \)

"Such that"

Curly bracket notation for set

Either \( S \in S \) or \( S \notin S \)

(Law of Excluded Middle)

\((P \lor \neg P)\)

If \( S \in S \) then, by def., \( S \in S \)

\( S \in S \) and \( S \notin S \)

a contradiction

If \( S \notin S \) then, by def., \( S \in S \)

\( S \in S \) and \( S \notin S \)

a contradiction

In fact, \( S \) is a class which is not a set