1. Is the following argument valid, whether or not you agree with the conclusion?

All of the students in my class are fierce or friendly. If a student is friendly then he or she is smiling. I can see students in my class who are not smiling. I conclude that my class contains fierce students.

2. Translate each of the following statements into well-formed formulae involving $P$ and $Q$ and implication. Now convert these into logically equivalent well-formed formulae that avoid implication (using only negation, conjunction and disjunction):

   (a) $P$ is a necessary condition for $Q$.
   (b) $P$ is a sufficient condition for $Q$.
   (c) $P$ is both necessary and sufficient for $Q$.
   *(d) $P$ is necessary but not sufficient for $Q$.
   *(e) $P$ is not necessary but is sufficient for $Q$.
   *(f) $P$ is neither necessary nor sufficient for $Q$.

3. Interpret “$P$ unless $Q$” using implication, and “$P$ unless and until $Q$” using an if and only if statement. Is the following true?

   “$P$ unless $Q$” if and only if “$Q$ unless $P$”.

4. Find formal proofs for the following sequents:

   (a) $P \Rightarrow (P \Rightarrow Q)$, $P \vdash Q$
   (b) $P \Rightarrow (Q \Rightarrow R)$, $\sim R$, $P \vdash \sim Q$
   (c) $\sim P \Rightarrow \sim Q$, $Q \vdash P$
   (d) $P \Rightarrow \sim Q \vdash Q \Rightarrow \sim P$
   (e) $P \Rightarrow Q$, $Q \Rightarrow R \vdash P \Rightarrow R$
   *(f) $P \Rightarrow (Q \Rightarrow R) \vdash (P \Rightarrow Q) \Rightarrow (P \Rightarrow R)$
   *(g) $P \Rightarrow (Q \Rightarrow (R \Rightarrow S)) \vdash R \Rightarrow (P \Rightarrow (Q \Rightarrow S))$
   *(h) $P \Rightarrow Q \vdash (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)$
   (i) $P \vdash (P \Rightarrow Q) \Rightarrow Q$
   *(j) $P \vdash (\sim (Q \Rightarrow R) \Rightarrow \sim P) \Rightarrow (\sim R \Rightarrow \sim Q)$
5. Prove that every wff (well-formed formula) has the same number of left brackets as right brackets.

*6. Let $W$ be any wff. Let $b(W)$ be the number of times a binary connective occurs and $p(W)$ the number of times propositional variables occur in $W$. Prove that

$$p(W) = b(W) + 1.$$ 

*7. Explain why the function

$$f : \mathbb{Z}^+ \to \mathbb{R}\setminus\mathbb{Q} \text{ where } f(x) = x + \sqrt{2}$$

is sensibly defined and injective. Why is $f$ not bijective?

*8. Recall in lectures we created a bijective map (a listing) $g : \mathbb{Z}^+ \to \mathbb{Q}$. Use $g$ and the map $f$ from the previous exercise to build a bijection $h : \mathbb{R}\setminus\mathbb{Q} \to \mathbb{R}$. Thus $|\mathbb{R}| = |\mathbb{R}\setminus\mathbb{Q}|$. 