

*Welcome to* **MATH3065 (LOGIC & FOUNDATIONS)**, Semester 1, 2011.

**Course Outline:** The course is in three parts:

**1. Turing Machines; 2. Logical Systems; 3. Zermelo-Fraenkel Set Theory.**

Part 1, focuses on abstract computation and its limits. After discussing what might be meant by a *Routine Computational Procedure* (RCP), we introduce the concept of a *Turing Machine*, and discover that, though very simple, Turing machines are capable in theory of simulating much of the routine computational work which seems to play a large part in mathematics. However, we find that there are certain mathematical *tasks* which cannot be performed by a Turing machine, nor by any finite state machine we seem to be able to imagine. We show that the famous “Blank Tape Halting Problem” (BTHP) is one of these “Turing impossible” tasks.

In Part 2 we study *Logical Systems* and discuss whether it is possible to set up such a system in such a way that it becomes *routine* to discover *and prove* all mathematical truths and only mathematical truths. We find that, because of the BTHP, there is no way such a system can be set up, if it is to include any substantial mathematics (such as elementary number theory). We climax this part with a discussion of one of the most dramatic discoveries of 20th century mathematics, Gödel’s Incompleteness Theorem, and its relationship with the BTHP.

Part 3 focuses on a particular logical system: Zermelo-Fraenkel Set Theory. This theory, which can be written down on one page, provides a foundation sufficient for all of mathematics. This section of the course will carry the development of mathematics within the system as far as construction of the real and complex numbers.

**Printed Notes: Logic and Foundations, 2011**, by H.M.Gastineau-Hills, (new revised edition). This set of printed notes includes all the tutorial exercises. It may be purchased in advance at KOPY STOP, 55 Mountain Street, Broadway.

**Lectures:** Three per week.

**Tutorials:** Twelve, one per week, starting week 2. Participation marks may be earned, one per week, to a maximum of 10. Each week it will be announced which of the exercises listed in the printed notes are to be discussed in the following tutorial. Each week you should study the relevant tutorial exercises *in advance*, and bring them to the tutorial (don’t expect your tutor to hand out copies of the exercises at the tutorial!). After each week tutorial solutions will be posted on the Web (see below).

**Assignments:** Two, each marked out of 5. The due dates will be announced well in advance (lateness penalty normally 1 mark out of 5 per day or part thereof!). After each assignment has been marked, solutions will be posted on the Web (see below).

**Consultations:** *Friendly consultation hour* times will be announced early in the semester. You are *most welcome* to attend these. They are naturally most heavily attended just before assignments are due, but you should take advantage of slack times to clear up difficulties in understanding lectures and tutorial solutions.

**The Web:** Important notices and material such as tutorial solutions will regularly be put on the Web. To access them all you will need an acrobat reader (freely available!). Please visit <http://www.maths.usyd.edu.au/u/UG/SM/MATH3065>

**Assessment:** 80% exam + 10% assignments + 10% tutorial participation.

*Best wishes,*      *Humphrey Gastineau-Hills (Carslaw room 525)*