

LOGIC & FOUNDATIONS TUTORIAL 1

1. Construct a TM (Turing machine) which performs the following *task*:
*Given an initial tape which is all blank except for a single * on some unknown square, to locate the *. (In effect, the TM must halt while scanning a *, with no other *s on the final tape.)*
Can you modify this TM so that the halting tape is all blank except for the scanned *?
2. (i) Some TMs never halt, whatever valid initial tapes they start on. Construct such a machine. (A “valid” tape contains only symbols from the TM’s alphabet, with only finitely many non-blank squares.)
(ii) Show that if a TM whose alphabet contains more than just the blank eventually halts when started on one valid initial tape, then there are infinitely many different valid initial tapes on which it will eventually halt.
3. Construct a TM to perform the following *task*, or prove that no TM can perform this task:
*Given an initial tape containing a finite number of *s scattered on it, and blanks elsewhere, to count the number of *s, that is, to generate a string of 1s which has as many 1s in it as there were originally *s on the tape then halt scanning one of these 1s.*
4. Describe a task which can be performed by a TM, but cannot be performed by any TM with only one non-halting state (regardless of how many symbols are in its alphabet).
5. In this exercise we consider TMs which are designed to be started on initial tapes of the form
$$\dots bb\underline{1}1\dots 1bb\dots$$
where there are a finite number of consecutive 1s, n say, the first of which is scanned, and blanks elsewhere. (If $n = 0$, take this as meaning $\dots bb\underline{b}bb\dots$.)
(i) Design a TM which, for any n , if started on an initial tape of the above form with n consecutive 1s, eventually halts scanning one of a string of $2n$ consecutive 1s. (We say of such a TM that for input n it outputs $2n$.)
(ii) As for (i), but outputting n^2 .
6. A certain TM will, for any n , if started on a tape $\dots bb\underline{1}1\dots 1bb\dots$ (with n 1s) eventually halt after an unknown number of steps. You, the user, wish to be able, for any n , to record the state of the final tape (by writing out symbols from a string of consecutive squares long enough to include the final scanned square and all non-blank squares). Is there a problem? Can you solve this problem by modifying the TM so it still does essentially the same task?